Computer Aided Design of Asymmetric Gear

*Vedang Singh and S. Senthilvelan**
Department of Mechanical Engineering, Indian Institute of Technology Guwahati
Guwahati 781039 INDIA
**Corresponding author (Email: ssvelan@iitg.ernet.in)*

Abstract

Gear design becomes inevitable for the development of any mechanical system and it necessitates considerable expertise. Due to the evolution of new materials and manufacturing processes, the utilization of asymmetric gears increases in recent years. Designers will be able to develop gear drives to handle larger torque with lesser noise and vibrations with the asymmetric profile. However the existing design procedure of symmetric gears does not holds good for these asymmetric gears. Hence by modifying tooth form factor, stress concentration factor, and load sharing factor, a MATLAB® code has been developed for designing asymmetric spur gears. Pressure angle limits were initially arrived by considering the gear tooth peaking and law of gearing. With in the range of pressure angle values, the asymmetric gear teeth which experiences minimum bending and contact stresses for the given loading condition can be chosen. In addition with the known number of tooth, module, and pressure angles of driving and coast side, asymmetric gear tooth coordinates were obtained which can be directly used for actual gear manufacturing

Keywords: Asymmetric gear design and expert system.

1. Introduction

In a standard symmetric gear, both left and right sides of a gear tooth profile have same bending and contact strength. However, in most practical cases, both the forward and backward rotations are not always used for power transmission [1-2]. Therefore, two sides of the gear tooth are functionally different for most gears. Even if one side (drive side) is significantly loaded for longer periods, the opposite side (coast side) is unloaded or slightly loaded for short duration only. Thus Asymmetric tooth are well suited for cases where the torque is transmitted mainly, in one direction.

In the recent years, wide range of advanced engineering materials is being increasingly used for gear manufacturing. Some of the most common materials are tailor modified metallic powders and fiber reinforced thermoplastics [3]. These materials are being processed via powder metallurgy processes and injection molding processes respectively. In both the category of materials, gears are not manufactured through metal removal process, by utilizing dies; parts are manufactured to the desired precision level in a single stage. Due to the nature of sintering and molding process, the surface fatigue strength of these basic materials is found to be superior [4]. Hence bending strength plays a predominant role in the gear design than contact strength in sintering and molding gears.

In the conventional symmetric gear design, bending strength of a particular gear material is improved by increasing the gear pressure angle. However when the pressure angle is increased at drive and coast side, both bending and contact strength increases, which is not necessary for sintering and molding gears. Hence by providing low pressure angle profile (standard 20°) for the drive side and high pressure angle profile for the coast side, only bending strength can be improved without improving contact strength [5] Hence gear mass, noise and vibration level can be significantly reduced. Similarly by providing standard pressure angle at coast side and higher pressure angle at drive side, both bending and contact strength can be significantly improved with less mass.

Litvin et al [6] proposed a modified geometry to localize and stabilize the bearing contact and obtain transmission errors of reduced magnitude. Simulation of meshing and contact of misaligned asymmetric spur gear has been developed. Stress analysis by finite element analysis has been accomplished for asymmetric and symmetric spur gear drives. It is confirmed that the contact and bending stresses for the driving side (with a larger pressure angle) of an asymmetric spur gear drives are reduced. Kapelevich [7] presented a method of research and design of gears with asymmetric teeth that enables to increase load capacity, reduce weight, size and vibration level. Two sets of asymmetric gears were manufactured in order to test vibration level of both high and low pressure angle drive sides for comparison. Asymmetric gears with high pressure angle have shown significantly reduced vibration level due to its low specific sliding ratio and low mesh stiffness due to the appropriate selection of coast pressure angle and fillet shape.

2. Asymmetric Gear Design

Symmetric gears are being designed with standard procedures, whereas in the asymmetric gears, decision of suitable pressure angles at coast side and drive side are more crucial. Though the increase in pressure angle increases bending strength, there is an upper limit of the pressure angle to avoid tooth peaking and to have safe contact ratio [8]. In addition, decision of gear tooth fillet radius also influences the upper limit of pressure angle. Hence in the design of asymmetric gears, these factors are to be considered in addition to the symmetric gear design procedures. Decision on pressure angle alters many crucial factors such as critical width, tooth form factor, stress concentration factor, load sharing factor, and magnitude of moment acting on a single tooth [9-10]. Hence there is a need for an expert system to predict the above said crucial factors and design an asymmetric gear possessing required bending and contact strength for a particular specific application.

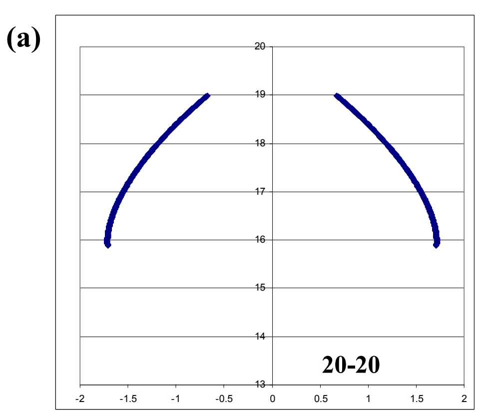
2.1 Gear tooth peaking

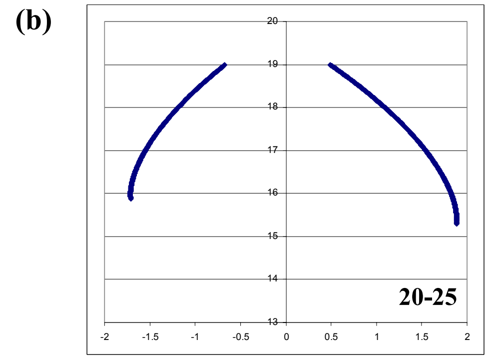
Major limitations of increasing the pressure angle is the reduction of gear tooth thickness at addendum circle. Tooth shape becomes more and more pointed as the pressure angle increases. Consequently the top land becomes correspondingly smaller and ultimately results in pointed tip. This phenomenon is termed as 'peaking'. The peaking limit sets a boundary to the maximum magnitude of pressure angle. Gear standard procedures such as IS, recommended that the tip thickness should be greater than equal to 0.2 times the module for the hardened gears. On the other hand, tooth flank becomes more curved as the pressure angle increases.

Figure 1a and 1b schematically indicate the change in flank width and top land due to the increase in pressure angle on one side. Figure 1c shows the decrease in top land width with increase in pressure angle up to the upper limit. The results obtained are for the following parameters, module =2, number of teeth in the pinion and gear = 17 and pressure angle on coast side = 20°.The limit of the tooth thickness on addendum circle is given as

$$S_a = r_a\left(\frac{\pi}{z} + \left(inv\alpha_c + inv\alpha_d\right) - \left(inv\alpha_{ac} + inv\alpha_{ad}\right)\right) \geq 0.2\text{m} \quad (1)$$

In the design of asymmetric gears, tooth peaking sets the upper limit of increasing pressure angle on either drive and coast side. Having a specified value of gear tooth fillet radius also limits the amount of pressure angle at both drive and coast side. However in this work, gear tooth fillet radius has not been considered. In actual case, root fillet radius significantly influences the gear tooth bending stress [11-12].





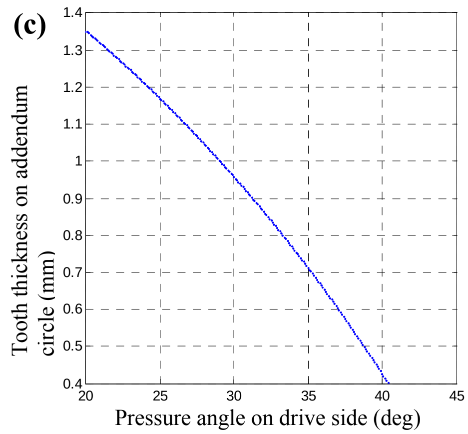


Figure 1: Influence of pressure angle on tooth width at top and land and flank region

2.2 Contact ratio

Increase in pressure angle on drive side profile may lower the limiting number of teeth to avoid undercutting; however the contact ratio of a gear pair significantly reduces. Contact ratio is a measure of the average number of teeth in contact during the period in which a tooth comes and goes out of contact with the mating gear. Due to this behavior, uniform transmission of angular motion may not be possible.

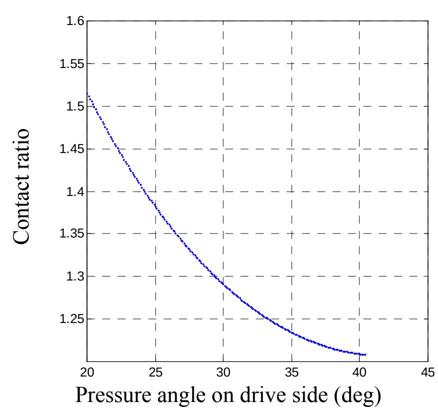


Figure 2: Influence of drive side pressure angle over contact ratio

Figure 2 indicates that behavior of contact ratio with the increase in pressure angle. The following contact ratio results, are obtained for module =2, number of teeth in the pinion and gear = 17 and pressure angle on coast side = 20°. The increase in pressure angle is limited when the contact ratio falls below 1.1. Beyond this value, the loading period of a single gear tooth pair significantly increases, which is undesirable under cyclic loading conditions. In addition, the highest point of single tooth contact (HPSTC) comes closer to the tooth tip, which is also undesired. Thus there is a stopping criterion for the drive side pressure angle i.e. it can only reach a specific maximum for a particular coast side pressure angle for normal operation. Limit of the contact ratio is given as

$$CR = \frac{\sqrt{r_{a1}^2 - r_{b1}^2} + \sqrt{r_{a2}^2 - r_{b2}^2} - a_d.\sin\alpha_d}{p.\cos\alpha_d} \geq 1.1 \qquad (2)$$

3. Gear Tooth Bending Stress

According to the ISO and DIN procedures, the maximum bending stress at the gear tooth root may be expressed as

$$\sigma_b = \frac{F_t}{L\,m} Y_{Fa} Y_{Sa} Y_{\varepsilon} Y_{\beta} \left(K_A K_V K_{F\beta} K_{F\alpha}\right) \qquad (3)$$

The tooth asymmetry has no impact on either overload factors $K_A, K_V, K_{F\beta}, K_{F\alpha}$ or the corrective factors Y_ε and Y_β [13]. Y_ε and Y_β are load sharing and contact ratio factor. For spur gears, contact ratio factor is unity and load sharing factor is given by

$$Y_\varepsilon = 0.25 + \frac{0.75}{CR}$$

Bending stress in asymmetric teeth for same torque, face width and module differs significantly from the bending stress in symmetric teeth because of the different values of form factor (Y_{Fa}) and stress concentration factor(Y_{Sa}).In teeth, the form and stress concentration factors are determined through the following relations as

$$Y_{Fa} = \frac{6\left(h_{Fa}/m\right)\cos\alpha_{Fan}}{\left(S_{Fn}/m\right)^2} \qquad (4)$$

$$Y_{Sa} = \left(1.2 + 1.3.\frac{S_{Fn}}{h_{Fa}}\right)\left(\frac{S_{Fn}}{2.\rho_F}\right)^{1/(1.21+2.3)(h_{Fa}/S_{Fn})} \qquad (5)$$

where

S_{Fn} Tooth thickness at critical section in root region

h_{Fa} Distance between the critical section and the point of intersection between the tooth axis and the direction of the force of contact

ρ_F Profile curvature radius at the critical section

It is clear from above equations that the form and notch factors are functions of S_{Fn}, h_{Fa} and ρ_F. Tooth thickness at critical section of asymmetric teeth plays a important role for reducing the bending stresses and also due to change of S_{Fn} and geometry, the distance between the critical section and point of intersection between tooth axis and direction of force of contact (h_{Fa}) also reduces. Tooth thickness at critical section for the entire possible asymmetric gear model were found out geometrically (shown in table 1) and these values were made use to arrive appropriate equation to compute critical section in the expert system.

Table-1: Critical tooth section thickness

Asymmetric Model	*Tooth Thickness at Critical Section (mm)*
20-20	3.201
21-20	3.239
22-20	3.280
23-20	3.325
24-20	3.372
25-20	3.428
26-20	3.485
27-20	3.552
28-20	3.620

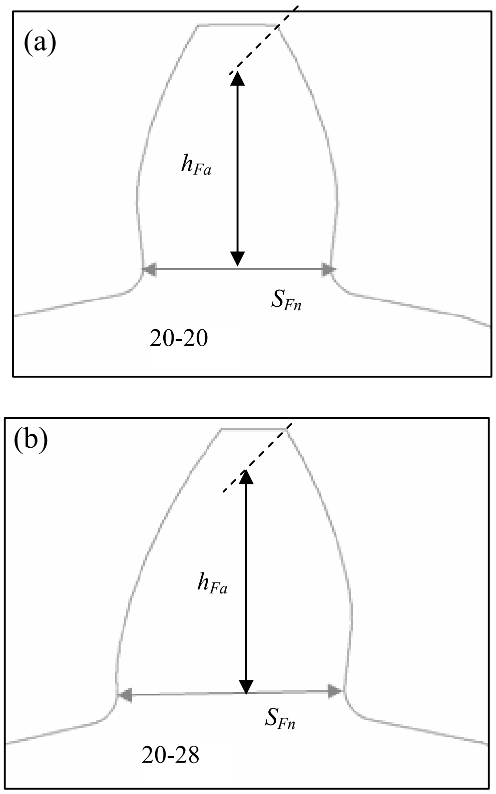


Figure 3: Critical sections for 20-20 and 20-28 asymmetric gear teeth

Figure 4 indicates the influence of pressure angle over tooth form and stress concentration factor. Tooth form factor increases with the increase in drive side pressure angle. This behavior is due to the increase in distance between the critical section and the position of contact force (h_{Fa} as shown in figure 3). However stress concentration factor significantly decreases with the increase in pressure angle, this behavior is due to the increase in flank curve (shown in figure 3)

Figure 5 indicates that the increase in drive side pressure angle significantly reduces the gear tooth bending stress. Thus by making use of the developed code, the designer can choose the desired asymmetric gear pair for the required application by considering the mechanical behavior of the chosen material. Figure 6 shows the algorithm used to compute the gear tooth bending stress.

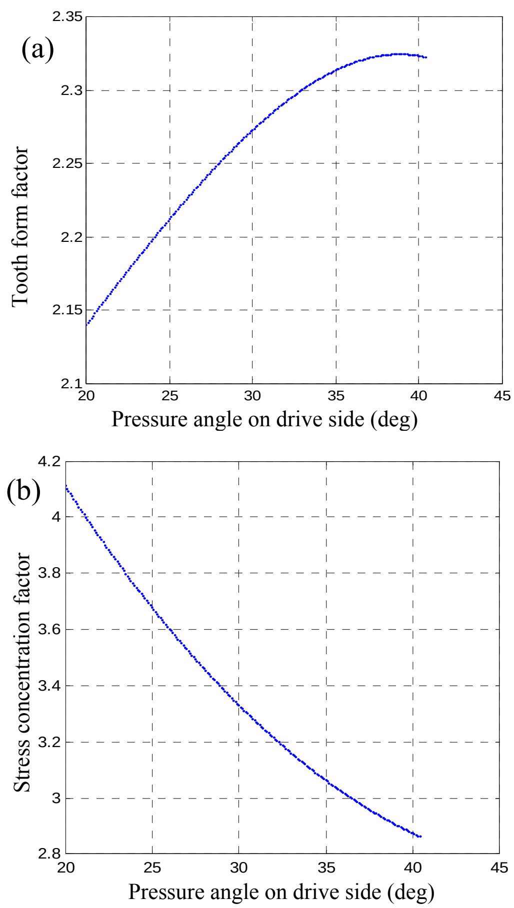


Figure 4: Behavior of factors over drive side pressure angle (a) tooth form factor and (b) stress concentration factor

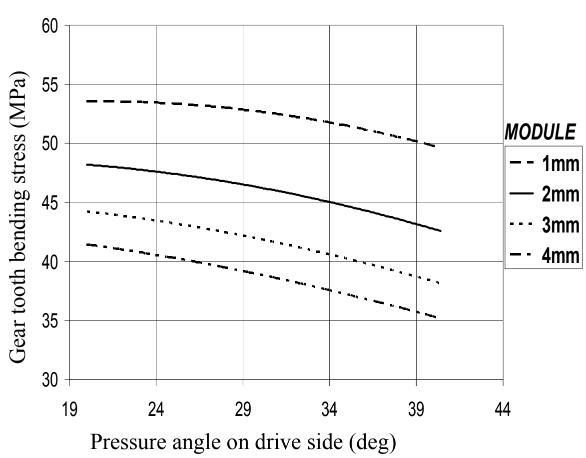


Figure 5: Influence of drive side pressure angle on gear tooth bending stress

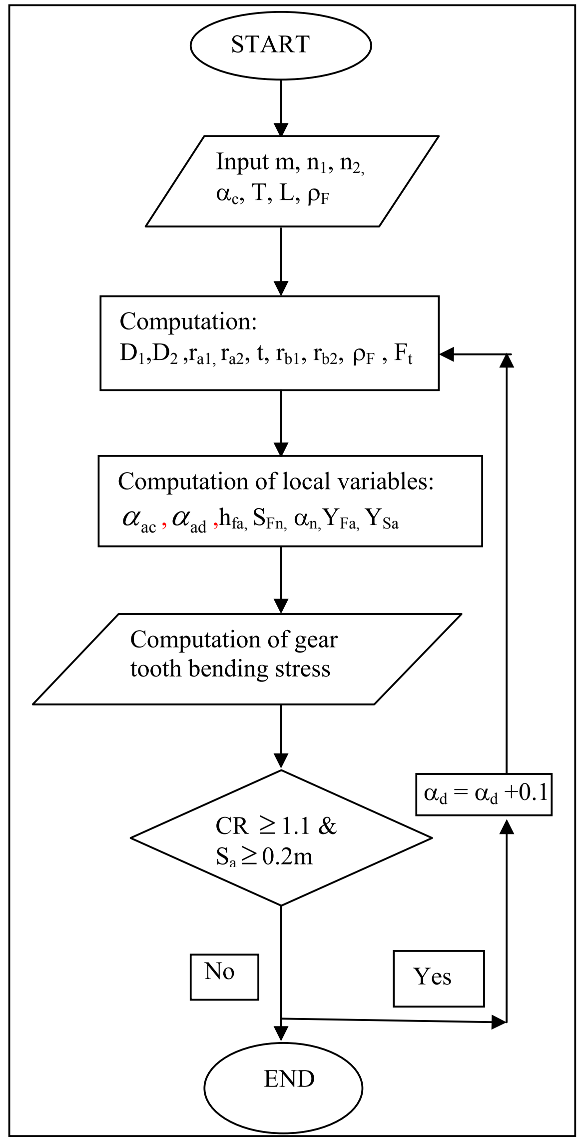


Figure 6: Flow chart to compute gear tooth bending stress

4. Gear Tooth Contact Stress

Contact fatigue strength of a gear material plays a major role in deciding the service life of gears. Some of the important influencing parameters that affect contact fatigue strength of a gear material are contact width, contact pressure distribution, and sub surface shear stress distribution [14]. Using Hertzian equation, contact width was computed using Poisson's ratio and Young's modulus of the test gear materials. The nature of asymmetric gear tooth contact surface is analogous to that of two contacting cylinders of constantly changing radius of curvature.

Figure 7 shows the equivalent cylinders of the gear tooth in contact at pitch point. For the test torque of 1.5 Nm, the contact stress and shear stress distributions were studied. When circular cylinders are held in contact by forces, a band of certain width along the tooth width is developed due to the mutual compressive pressure. As the tooth surfaces move relative to each other with a combination of rolling and sliding, the contact band continuously moves. Hence the stress pattern developed within this band is complicated. When the gear tooth surfaces are in contact, the maximum compressive stress is generated at the center of the contact region. Below the contact region, the maximum sub surface shear stresses are developed. Hertz developed equations for the stresses, which are generated when two curved surfaces are in contact. Beside contact pressure; sliding velocity, lubricant viscosity and frictional forces also influence the stress distribution.

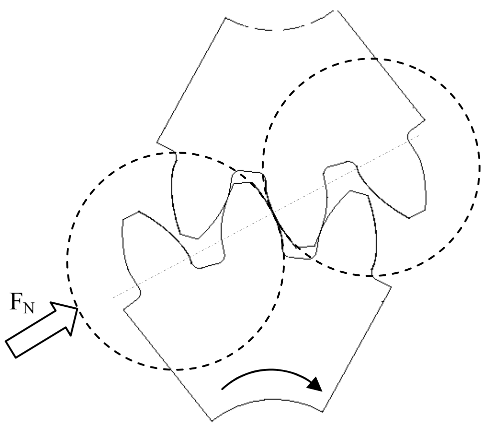


Figure 7: Schematic representation of equivalent cylinders from the pair of asymmetric gear tooth in mating position at pitch point.

When the curved surfaces pressed against each other, an contact area of width, b and length, L (gear face width) develops. The stress distribution is elliptical across the width. The maximum contact pressure, $p_{(\max)}$ is given by

$$p_{\max} = \frac{4F_N}{\pi bL} \tag{6}$$

where contact width, b is given by

$$b = \sqrt{\frac{8F}{\pi L}\frac{(1-\nu_1^2)/E_1 + (1-\nu_2^2)/E_2}{1/D_1 + 1/D_2}} \tag{7}$$

where F is the applied force, ν_1 and ν_2 are the Poisson's ratios of the two materials, D_1 and D_2 are equivalent cylinder diameters & E1 and E_2 are the respective moduli of elasticity.

The equivalent diameters of gear tooth at the pitch point are given by

$$D_1 = d_1 \mathrm{Sin}\alpha_d \text{ and } D_2 = d_2 \mathrm{Sin}\alpha_d \tag{8}$$

where α_d is drive side pressure angle at pitch circle, $d_1$and d_2 are pitch circle diameters of pinion and gear respectively.

$$F_N = \frac{2T/d_1}{\cos\alpha_d} \tag{9}$$

where F_N is the normal force, and T is the transmitted torque

The principal stresses (σ_x, σ_y and σ_z) at various depths (z) are computed using Smith - Liu equations.

$$\sigma_x = -2\nu p_{\max}\left[\sqrt{1+\frac{z^2}{b^2}} - \frac{z}{b}\right] \tag{10}$$

$$\sigma_y = -p_{\max}\left[\left(2 - \frac{1}{1+\frac{z^2}{b^2}}\right)\sqrt{1+\frac{z}{b^2}} - 2\frac{z}{b}\right] \tag{11}$$

$$\sigma_z = \frac{-p_{\max}}{\sqrt{1+z^2/b^2}} \tag{12}$$

From the principle stresses, shear stress,τ, is computed using following equation

$$\tau = \frac{1}{2}\left[\sigma_x - \sigma_y\right] \tag{13}$$

Figure 8 shows the sub surface shear stress profile of the investigated asymmetric gear pair. The maximum shear stress is responsible for the surface fatigue failure of the contacting surfaces. The crack originates at the point of maximum shear stress below the surface and progress to the surface.

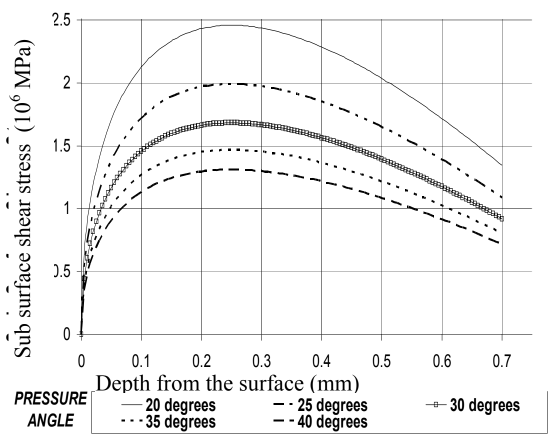


Figure 8: Profile plot of maximum sub surface shear stress below the surface for various drive side pressure angle

From the figure 8, it is infer that maximum shear stress occurs nearly at 0.25 mm depth from the surface irrespective of the drive side pressure angle. Hence for both symmetric and asymmetric gears, the type of hardening and depth at which hardness is required will not be different. However, the maximum shear stress decreases when the drive side pressure increases. Thus the increase in drive side pressure angle of asymmetric gear increases the contact fatigue life, for the given loading conditions.

5. Asymmetric Gear Geometry

The change in tooth profile from symmetric to asymmetric imposes difficulty in manufacturing through conventional metal removal process. Computer numerically controlled processes such as wire electric discharge machining process makes it possible to obtain this type of complex profile. However coordinates of complex asymmetric gear tooth are necessary to feed the manufacturing systems. Developed expert system provides these data for manufacturing sector. Once the asymmetric gear design for

a particular application is completed, parameters such as module, the number of teeth in the pinion and the pressure angles of the tooth on the coast and drive sides can be fed into the system and coordinates for the face and flank portion of asymmetric gear teeth can be obtained. Then obtained coordinates can be fed into any modeling software which provides complete gear model and its coordinates. The following flow chart (figure 10) states how to compute the co-ordinates (in polar form) of all the points on a gear tooth:
The intermediate variables are:

$\alpha_i = \cos^{-1}(r_{b1}/r)$ where r is the radial distance of the point from the centre of the pinion.

Involute of α_i is given by:

$inv(\alpha_i) = \tan(\alpha_i) - \alpha_i$; $\theta = (t/(2*r)) + Inv(\alpha_d)$;

$\theta_i = \theta - inv(\alpha_i)$;

The final co-ordinates are: $x = r*\sin(\theta_i)$; $y = r*\cos(\theta_i)$

Computation of the pitch circle radius, addendum circle radius, base circle radius for the coast side, base circle radius for the drive side, circular pitch of the tooth and tooth thickness along pitch circle were done to obtain the complete geometrical model of asymmetric gear.

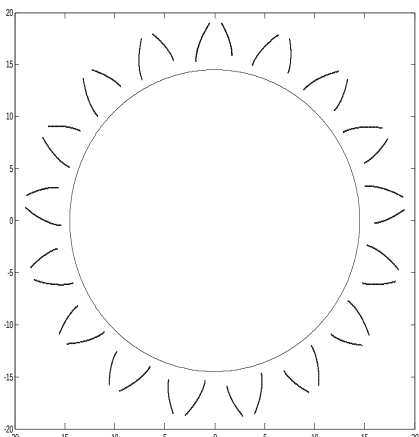


Figure 9a: Gear model obtained from MATLAB

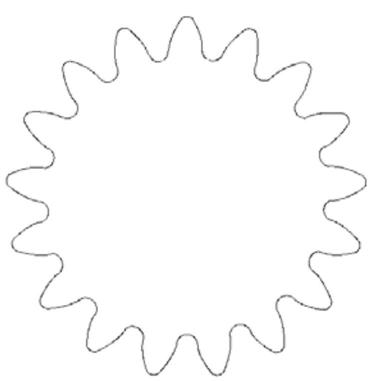


Figure 9b: Complete asymmetric gear model

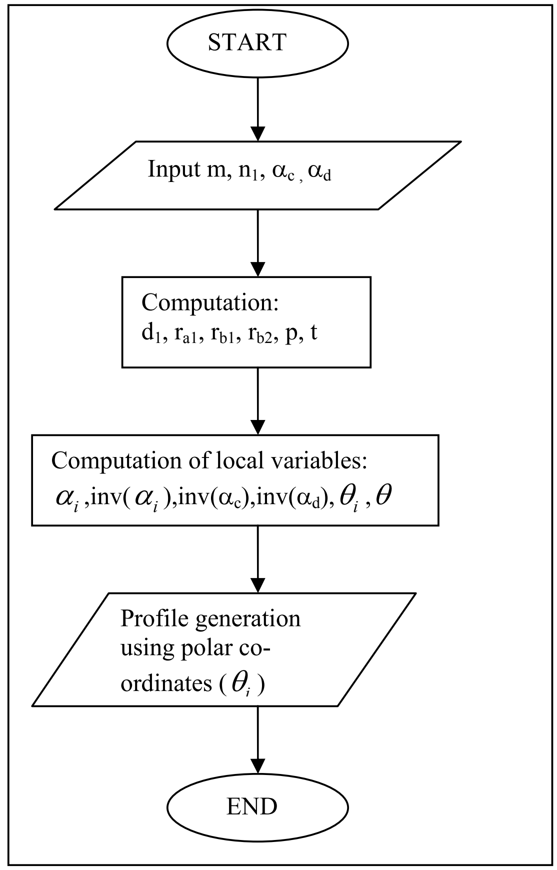


Figure 10: Flow chart to obtain asymmetric gear coordinates

Figure (9a) is obtained from MATLAB for a gear of having 17 numbers of teeth with coast side 20^0 and drive side 25^0. In which top land and fillet region of gear is not completed. By fixing the gear tooth fillet radius as 0.25 times the module, complete asymmetric gear model has been obtained using AutoCAD ® as shown in figure 9b.

6. Conclusions

- Design of asymmetric gear necessitates the modifications of existing symmetric gear design procedures.
- Decision on maximum magnitude of drive side/coast side pressure angle is constraint by the safe contact ratio and tooth peaking effect.
- Asymmetric gears with higher pressure angle on drive side, posses increased bending and contact strength but with reduced weight
- An expert system has been developed to compute, form factor and stress concentration factor, bending stress and sub surface shear stress for the specified module, no. of teeth on pinion and gear, coast and drive side pressure angle for a specific application.

Nomenclatures

α_c	Pressure angle on the coast side
α_d	Pressure angle on the drive side
α_{ac}	Pressure angle on the coast side at addendum
α_{ad}	Pressure angle on the drive side at addendum
α_{an}	Angle between the direction of the force of contact(applied at the outside radius) and the normal at the tooth axis
σ_x, σ_y, σ_z	The principal stresses
τ,	Shear stress
ρ_F	Radius of the fillet
ν_1, ν_2	Poisson ratio of the pinion and gear
CR	Contact ratio
d_1, d_2	Pitch circle diameter of pinion and gear
E_1, E_2	Modulus of elasticity of the material of the pinion and gear
F_r	Radial component of normal tooth load
F_t	Tangential load at the pitch circumference
F_N	Normal force
h_{Fa}	Distance from critical section to intersection of the tooth centerline and line of action for load at tip of tooth
L	Face width
m	Module of the gear
n_1, n_2	Number of teeth in pinion and gear
r_{a1}, r_{a2}	Addendum circle radius of pinion and
r_{b1}	Base circle radius for the coast side
r_{b2}	Base circle radius for the drive side
p	Circular pitch of the tooth
S_a	Tooth thickness at addendum circle
S_{Fn}	Tooth thickness at critical section
t	Tooth thickness along pitch circle
T	Torque
Y_{Fa}	Form factor
Y_{Sa}	Stress concentration factor
Y_β	Contact ratio factor
Y_ε	Load sharing factor

Acknowledgement

Authors wish to thank Mr Walsange Ganesh, MTech Student for providing FEA results of asymmetric gears which were used to validate the developed expert system.

References

[1] G. Francesco and S. Marini, "Structural Analysis of Asymmetric Teeth : Reduction of Size and Weight", *Gear Technology*, Sept/Oct 1997.

[2] G. Deng and N. Tsutomu, "Enhancement of Bending Load Carrying Capacity of Gears using an Asymmetric Involute Tooth", *JSME International Conference on Motion and Transmissions*, Fukuoka, Japan, 2001.

[3] R.E. Kleiss, A.L. Kapelevich, and N.J. Kleiss () "New opportunities with molded gears". *AGMA Fall Technical Meeting*, Detroit, 2001.

[4] K. Cavdar, F. Karpat and F. Babalik, "Computer Aided Analysis of Bending Strength of Involute Spur Gears with Asymmetric Profile, *Journal of Mechanical Design*, Vol. 127, No 3, 2005, pp. 477– 484.

[5] J. E. Shigley and C. R. Mischke, *Mechanical Engineering Design*, Tata McGraw-Hill Publishing Company Limited, 2001.

[6] G. Deng and T. Nakanishi, "Bending Load Capacity Enhancement using an Asymmetric Tooth Profile", *Japan Society of Mechanical Engineer's International Journal*, Vol. 46, No. 3, 2003, pp. 1171–1177.

[7] F. L. Litvin and A. L. Kapelevich, "Asymmetric Modified Spur Gear Drives- Reduction of Noise, Localization of Contact, Simulation of Meshing and Stress Analysis", *Computer Methods in Applied Mechanics and Engineering*, Vol. 188, 2000, pp. 363–390.

[8] L. Kapelevich, "Geometry and Design of Involute Spur Gears with Asymmetric Teeth", *Mechanism and Machine Theory*, Vol. 35, No. 1, 2000, pp. 117–130.

[9] G. M. Maitra, *Handbook of Gear Design*, Second Edition, Tata McGraw-Hill Publishing Company Limited, 2005.

[10] G. D. Francesco and S. Marini, "Asymmetric Teeth – Bending Stress Calculation", Gear Technology, March / April 2007, pp. 52–55.

[11] H. E. Merritt, *Gear Engineering*, Third Indian Edition, A. H. Wheeler & Company Limited, 1992.

[12] V. Spitas and C. Spitas, "Increasing the Strength of Standard Involute Gear Teeth with Novel Circular Root Fillet Design", *American Journal of Applied Sciences* Vol. 2, No. 6, 2005, pp. 1058–1064.

[13] S. Senthilvelan and R. Gnanamoorthy, "Effect of Gear Tooth Fillet Radius on the Performance of Injection Molded Nylon 6/6 Gears", *Journal of Materials & Design*, Vol. 27, No. 8, 2006, pp. 632–639.

[14] S. Senthilvelan and R. Gnanamoorthy, "Surface Failure Analysis of Unreinforced ,Glass and Carbon Fiber Reinforced Nylon 6/6 Spur Gears", *International Journal of Plastics Technology*, Vol. 9, 2005, 444-452.