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Engineering mechanical systems at micron level-computational and modeling paradigms

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Abstract

Engineering incredibly complex systems at the micron and sub-micron scales is a new fascination of this century. With increasing application possibilities fast and accurate simulations of such systems are a prime concern for saving time and expenditure involved in design, fabrication and testing. Simulating systems at this level open up very exciting avenues of research particularly due to the confluence of multiple domain physics. The present paper explores some of the exciting advances in the area of simulation of micron level devices and an example of the dynamics of a coupled electro-mechanical system using a coupled FEM-BEM approach.

Keywords: Microelectromechanical systems, fast-computation, computational mechanics, multi-physics.

1 Introduction

The state of the art of microelectromechanical systems is poised for a great leap forward in the coming decade with increasingly better control and understanding of multi-physics interaction at the micron and submicron level. Devices like microfuel cells, digital micromirror displays (DMD), micromotors, radio frequency switches and efficient energy harvesters along with greater MEMS-CMOS integration are fueling a new huge market [1]. The market of MEMS is projected to grow upto $95 billion by the begining of the next decade making it one of the most lucrative areas of research and investments [1].

This paper deals with the simulation aspect of MEMS devices. Making a MEMS device can involve an array of complex and expensive fabrication processes. Simulation is an essential tool for proper design to prevent expensive waste of manufacturing effort and also a safeguard against usage failure. An archetypical MEMS vibrator is discussed and a coupled FEM-BEM multiphysics approach is explained. A few numerical results generated from the method are also presented. Finally, the fast multipole method (FMM) and its promise for MEMS simulation is discussed in a brief section.

2 Modeling MEMS

Modeling of a MEMS is a vital first step for design or testing. Modeling incorporates a wide range of phenomena and their mutual interactions. A plethora of force domains may interact with each other in a single device requiring simultaneous modeling of electrostatic, elastic, fluid, magnetic or even chemical fields. This multiphysics interaction results in a spectrum of rich behavior in MEMS and at the same time makes the task of modeling very challenging.

2.1 Electro-Mechanical Modeling of MEMS

One of the early approaches to deal with MEMS modeling was the lumped parameter modeling. This model lumps physical properties of systems like mass, elastic-stiffness, capacitance and inductance into single idealized simulation elements. This modeling is valid as long wavelength of signals are greater than all characteristic lengths of the system. Transducers have been simulated using this model [3]. This method's applicability is improved by using a distributed parameter system [4] which can be used to introduce infinite degrees of freedom into the system. The usefulness of lumped parameter decreases for more complicated large deformation and nonlinear problems.

On the other hand, domain specific multiphysics simulations have been more popular in recent times for CAD purposes being very useful in their versatility. The primary domains that intersect in MEMS research are electromagnetic, mechano-fluidic, thermal and chemical. Even though MEMS are micron level systems, the continuum hypothesis is entirely valid[1]. Hence, all laws of elasticity can be used in a very classical sense for its analysis. However, it is imperative to understand and exploit the uniqueness of MEMS system. Most problems are resistant to analytic solutions and require some kind of numerical technique. The most widely used is the finite element method (FEM) though finite differences are also used widely. Ones the weak form has been derived the system is discretized and solved using

[1] Semiclassical methods can still find use in nanoscale NEMS devices although the continuum hypothesis is not entirely valid here.

standard nonlinear equation solving techniques. Mukherjee et.al.[2] have derived an explicit finite element formulation suitable for dynamics of MEMS beams and plates using a moderate deformation assumption. Another line of attack uses boundary element method (BEM) to tackle the elastic problem accelerated through fast multipole methods (FMM). However, this is still an active area of research and is yet to prove its superiority over the conventional finite element modeling. On the other hand, for the electrostatic domain, external BEM formulation especially with thin structure approximation [5] has been more profitable by eliminating the need for conventional BEM analysis over thin plates which can result in ill conditioned matrices. On the other hand Liu [7] has developed a novel composite BEM and Hypersingular BEM to solve the electrostatic problem without resorting to the thin structure approximation. An FEM approach to electrostatic problem is also used but for large number of MEMS devices with complicated spacings, BEM has an advantage of reduction of the geometric dimension of the problem by one thereby reducing the meshing overhead. BEM is also gaining an advantage in terms of computational speed because of the introduction of the FMM to be discussed briefly later.

2.2 Fluidic Effects in MEMS

Fluidic effects on MEMS is an active area of research spawning an enormous amount of literature in very short time. The field of microfluidics is entirely dedicated to the understanding of these processes and its discussion is beyond the scope of this present paper. However, the damping effects of fluids can have a direct bearing on the dynamics of a MEMS device and will be discussed here. Even though lumped parameter methods are used in fluidic systems, the domain specific approach leads to better understanding and more accurate solution. The governing equation for fluids is the Navier-Stokes equation. In the small length scales involved, the Navier-Stokes yields to simplified versions more amenable to solution. In general, a dichotomy exists in modeling damping effects of fluids in MEMS. One can assume a low Reynolds number (low excitation frequency) when the devices are moving relatively slowly and presume Stokes flow to be valid. The Stokes flow leads to the equation,

$$\nabla p(\mathbf{x}) - \mu \nabla^2 \mathbf{v}(\mathbf{x}) = \mathbf{0}, \qquad \mathbf{x} \in B \tag{1}$$

In the above, $\mathbf{v}$ is the velocity, p is the pressure and μ is the dynamic viscosity of the fluid. Also, B is the region *exterior* to the structure and. The stress tensor σ inside the fluid, and the surface traction τ on the solid surface, are defined by the equations:

$$\sigma(\mathbf{x}) = -p(\mathbf{x})\mathbf{I} + \mu[\nabla \mathbf{v}(\mathbf{x}) + \nabla^T \mathbf{v}(\mathbf{x})], \qquad \mathbf{x} \in B \tag{2}$$

$$\tau(\mathbf{x}) = \sigma(\mathbf{x}) \cdot \mathbf{n}(\mathbf{x}), \qquad \mathbf{x} \in \partial B \tag{3}$$

where $\mathbf{n}(\mathbf{x})$ is the unit normal to the surface. However, in many situations a fluid fills the space between two vibrating parallel plates. When the plate moves down, the fluid pressure increases and fluid is squeezed out from the edges of the plates. On the other hand, when the plate moves up, the pressure is eased and viscous drag of the fluid creates a dissipative mechanical force resulting in what is known as squeezed-film damping [8]. The squeezed film modeling is a now a rapidly developing area of research and many new kinds of MEMS are being successfully simulated assuming this model(See [9]).

2.3 Eulerian and Lagrangian Paradigms of Simulation

One of the classical ways of simulation of a coupled MEMS is to treat the domains separately, solving one and feeding the output as an input to the other while dealing with the current configuration of the system. This is the so called Eulerian iterative process. Iterative processes are the traditional way of solving the coupled electromechanics of the system. However, Aluru et.al. [10] have suggested a Newtonian scheme for solution of the coupled problem wherein the entire problem is solved *in toto*. In this scheme, the Eulerian approach runs into trouble because the need to compute derivatives of the changing domain in the electrostatic BEM part of the code. Aluru [10] has proposed solving the Lagrangian version of the coupled problem of this kind wherein all the variables are referred to the initial configuration [11]. This entirely eliminates the need to remesh the domain due to excessive deformation giving it an upper edge over other techniques. The Lagrangian-Newtonian framework also helps to easily integrate additional physical domains and inherits the excellent convergence properties of the Newton method.

3 Coupled FEM-BEM Analysis of Thin MEMS Beam

One can demonstrate the ideas developed in the previous section to carry out a numerical simulation of a MEMS beam clamped on both sides. Owing to its small size, significant forces and/or deformations can be obtained with the application of low voltages ($\approx$ 10 volts). Examples of devices that utilize vibrations of such beams are comb drives, synthetic microjets, microspeakers etc. The system is schematically described in Fig. (1). The electric, fluidic damping, inertial and mechanical restoring forces are shown respectively as F_E, F_D, F_I and F_M. The fluidic damping is not being considered in this paper but is fairly straightforward and easily integrable in the general scheme of the method and explained in short in the appendix.

The beam deforms when a potential V is applied between the two conductors. The charge redistributes on the surface of the deformed beam, thereby changing the electrical force on it and this causes the beam to deform further. As the deformation starts, the damping effects due to fluids come into play. The system then undergoes vibrations and the complete analysis of the system is done using the Newton scheme. Modeling of the entire system involves modeling

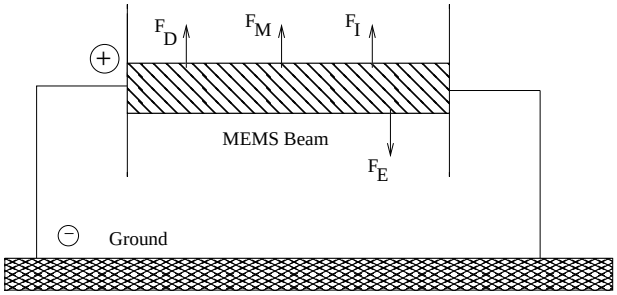

Figure 1: Configuration of the MEMS Beam

of three intersecting physical phenomena, electrostatic, mechanical and fluidic. Each needs a corresponding modeling technique for better efficiency and accuracy. The external electric field is modeled using the Lagrangian version of the thin beam BEM approach [12] together with a hypersingular post processing gradient BIE to find the individual charges. The mechanical problem is tackled using a moderately large deflection FEM analysis. Finally, a Newton scheme developed analogous to [10] is used to solve the entire coupled nonlinear problem.

3.1 Electrical Problem - BEM Formulation

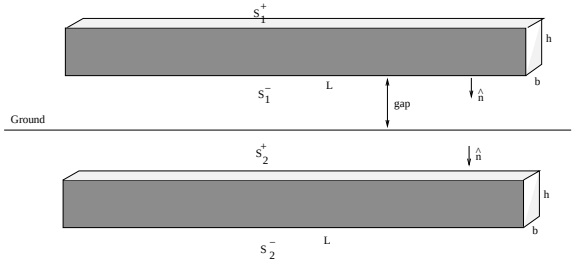

Figure 2: Equivalent electrostatic problem

The governing electrical equations can be derived using Laplace's equation applied to the electrical potential. Our system prototype described in Fig. (1) can be reduced to Fig. (2) using the method of images. The derivation of boundary integral equation for the external laplace problem using *thin structure assumption* yields the following equations [12]:

$$\phi(\mathbf{x}^+) = -\int_{s_1^+ \cup s_2^+} \frac{\log r(\mathbf{x}^+,\mathbf{y})\beta(\mathbf{y})}{2\pi\varepsilon} ds(\mathbf{y}) \qquad (4)$$

Where $\beta(\mathbf{x}) = \sigma(\mathbf{x}^+) + \sigma(\mathbf{x}^-)$, $\mathbf{x}^+ \in S_1^+, \mathbf{x}^- \in S_1^-$, $\mathbf{r}(\mathbf{x},\mathbf{y}) = \mathbf{y} - \mathbf{x}$ from Fig. (2) and ε is the permittivity of the medium. The lower beam being identical to the upper doesn't need separate analysis as charges are simply equal and opposite. It must be noted that the above equation incorporating the thin structure assumption does not contain information about the individual charges on the opposite surfaces. To obtain that information, one can use Eq. (4) to get the following hypersingular BEM equation [12],

$$\frac{1}{2}[\sigma(\mathbf{x}^+) - \sigma(\mathbf{x}^-)] = \int_{s_1^+ \cup s_2^+} \frac{\beta(\mathbf{y})\mathbf{r}(\mathbf{x}^+,\mathbf{y}) \cdot \mathbf{n}(\mathbf{x}^+)}{2\pi r^2(\mathbf{x}^+,\mathbf{y})} ds(\mathbf{y}) \qquad (5)$$

where $\mathbf{n}(\mathbf{x}^+)$ is the unit normal vector at point $\mathbf{x}^+$.

It must be noted that Eq. (4) and Eq. (5) contain singular kernels and must be regularized (See [5] and [12] for regularization version) before the BEM analysis can be carried out.

3.2 Fully Lagrangian Electrical BEM

Eq. (4) and Eq. (5) are the set of BEM equation required to solve the electric problem. However, they are still expressed in the current or Eulerian coordinates and need to be expressed in Lagrangian coordinates. The change of variables for the displacement vectors can be formulated as,

$$\begin{aligned} \mathbf{r}(\mathbf{x}(\mathbf{X}),\mathbf{y}(\mathbf{Y})) \equiv \mathbf{R}(\mathbf{X},\mathbf{Y}) &= \mathbf{y}(\mathbf{Y}) - \mathbf{x}(\mathbf{X}) \\ &= \mathbf{Y} + \mathbf{u}(\mathbf{Y}) - \mathbf{X} - \mathbf{u}(\mathbf{X}) \\ &= \mathbf{R_0}(\mathbf{X},\mathbf{Y}) + \mathbf{u}(\mathbf{Y}) - \mathbf{u}(\mathbf{X}) \end{aligned} \qquad (6)$$

Where $\mathbf{R_0}$ is the initial relative position. Next, define Σ, the charge density per unit undeformed surface area. Also define:

$$B = \Sigma^+ + \Sigma^- \qquad (7)$$

The transformation of Eq. (4) is pretty straightforward from the above transformation rules and noting that $\Sigma dS = \sigma ds$.

$$\phi(\mathbf{X}^+) = -\int_{S_1^+ \cup S_2^+} \frac{\log R(\mathbf{X}^+,\mathbf{Y})B(\mathbf{Y})}{2\pi\varepsilon} dS(\mathbf{Y}). \qquad (8)$$

where upper case letters denote Lagrangian description. However, we need more care to transform the hypersingular BEM Eq. (5). The conversion to the Lagrangian framework can be started by using Nanson's Law [17]:

$$\mathbf{n}ds = J\mathbf{N} \cdot \mathbf{F}^{-1} dS. \qquad (9)$$

where $\mathbf{n}$ and $\mathbf{N}$ are unit normal vectors to ∂b and ∂B, at the generic points $\mathbf{x}$ and $\mathbf{X}$, respectively, $\mathbf{F} = \frac{\partial \mathbf{x}}{\partial \mathbf{X}}$ is the deformation gradient, $J = det(F)$ and dS is an area element on ∂B. Here, $\mathbf{X}$ and $\mathbf{x}$ denote coordinates in the undeformed and deformed configurations, respectively. From, Eq. (9) and noting that $\Sigma dS = \sigma ds$ it follows that:

$$\Sigma = J\sigma|\mathbf{N} \cdot \mathbf{F}^{-1}|. \qquad (10)$$

4 Mechanical Problem in the Elastic Beam

Nonlinear deformation of a beam without any initial in-plane forces, is discussed in this section. The beam is linearly elastic, has immovable ends and is of uniform cross section. The cross section is symmetric such that there is no twisting of the beam under applied bending moments. Also, $u(x)$ is the axial deformation and $w(x)$ the transverse displacement of the mid-line of the beam.

4.1 The Model

The kinematic equations can be derived starting from the following nonlinear strain-displacement equation [13]:

$$\varepsilon_{ij} = \frac{1}{2}\Big(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\Big) + \frac{1}{2}\Big(\frac{\partial u_m}{\partial x_i} \cdot \frac{\partial u_m}{\partial x_j}\Big). \tag{11}$$

This leads to the kinematic equations:

$$\varepsilon_{xx} = u_{,x} + 1/2 \cdot (w_{,x})^2 \tag{12}$$
$$\kappa_x = -w_{,xx}, \tag{13}$$

here, ε_{xx} is the midline axial strain and κ_x is the curvature. Here $_{,x}$ denotes the derivative with respect to the axial coordinate x. The strain energy $\mathcal{E}^{(s)}$ and the kinetic energy $\mathcal{E}^{(k)}$ of an uniform beam of length L are,

$$\mathcal{E}^{(s)} = \frac{ES}{2}\int_0^L [(u_{,x})^2 + u_{,x}(w_{,x})^2 + (1/4)(w_{,x})^4]dx + \frac{EI}{2}\int_0^L (w_{,xx})^2 dx \tag{14}$$

$$\mathcal{E}^{(k)} = \frac{\rho S}{2}\int_0^L [(\dot{u})^2 + (\dot{w})^2]dx. \tag{15}$$

Here, E, ρ, L, S, L are the Youngs modulus, density (mass per unit volume), length, area of cross section, and area moment of inertia of the cross section of the beam, respectively, and a superposed dot denotes differentiation with respect to time t. Similarly the work expression can be written as,

$$\mathcal{W} = \int_0^L (H_x du + H_y dw + M dw_{,x})dx. \tag{16}$$

Here H_x, H_y and M are the axial force, transverse force and bending moment, respectively.

4.2 Finite Element Model for Beams with Immovable Ends

The procedure followed here for FEM discretization of vibrating beams, is similar to standard methods(see, e.g., Zienkiewicz and Taylor [14]). However, in this particular problem the standard beam element needs a slight modification. This modification is necessitated because the usual linear interpolation for the axial deformation results in discontinuities during residue computation in the Newton's scheme. Hence, a quadratic interpolation is taken for the axial deformation. A standard Hermitian interpolation is used for bending . The beam element used in this present problem thus has a total of seven degrees of freedom. Three axial at three axial nodes and two transverse and two rotational degrees of freedom at the end nodes. These degrees of freedom can be written as:

$$\mathbf{u} = [u_1\ u_2\ u_3] \qquad \mathbf{w} = [w_1\ w_2] \qquad \theta = [w_{,x1}\ w_{,x2}] \tag{17}$$

Now, the values of the primary deformations $\mathbf{u}$, $\mathbf{w}$ inside the elements can be interpolated from the above nodal values using,

$$\begin{bmatrix} u(x,t) \\ w(x,t) \end{bmatrix} = \begin{bmatrix} N^{(I)}(x) & 0 \\ 0 & N^{(0)}(x) \end{bmatrix} \cdot \begin{bmatrix} q^{(I)}(t) \\ q^{(O)}(t) \end{bmatrix} \tag{18}$$

wherein

$$[N^{(I)}(x)] = [N_1\ N_2\ N_3], \qquad [N^{(O)}] = [P_1\ P_2\ P_3\ P_4] \tag{19}$$

$$[q^{(I)}(t)] = [u_1\ u_2\ u_3]^T, \qquad [q^{(O)}(t)] = [w_1\ \theta_1\ w_2\ \theta_2] \tag{20}$$

Here N_k and P_k are third order Lagrange and cubic (Hermite polynomials) interpolation functions, respectively and $q^{(I)}$ and $q^{(O)}$ contain the appropriate nodal degrees-of-freedom. Substituting the interpolations from Eq. (18) into the work energy expressions from Eq. (14), Eq. (15) and Eq. (16) and use of Hamilton's principle leads to the following element level equations [2]:

$$[\mathbf{M}]\{\ddot{q}\} + [\mathbf{K}]\{q\} = \{P(t)\} \tag{21}$$

Here, $[M]$ is the consistent mass matrix, $[K]$ the stiffness matrix, $\{P(t)\}$ the work equivalent load vector and $\mathbf{q} = \{q\} = [q^{(I)} \quad q^{(O)}]$ (See [2] for details of these matrices and vectors). The loading is caused by the electrical charges and is given by the traction equation expressed in Lagrangian form using the transformation laws developed in section 3.2,

$$\mathbf{H} = -\frac{J\sigma^2 \mathbf{N}\cdot\mathbf{F}^{-1}}{2\varepsilon} = -\frac{\Sigma^2}{2J\varepsilon}\frac{\mathbf{N}\cdot\mathbf{F}^{-1}}{|\mathbf{N}\cdot\mathbf{F}^{-1}|} \tag{22}$$

5 Newton's Scheme for Solving the Coupled Problem

Newton's method is an iterative root-finding algorithm that uses the first few terms of the Taylor series of a function $f : \mathbb{R} \to \mathbb{R}$ in the vicinity of a suspected root. The algorithm can be written for a one dimensional case as,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \qquad n \geq 0.$$

For the multivariate case, $f : \mathbb{R}^p \to \mathbb{R}^p$,

$$x \in \mathbb{R}^p : f(x) = 0 \in \mathbb{R}^p$$

$$x_{n+1} = x_n - Jf(x_n)^{-1} f(x_n), \qquad n \geq 0 \tag{23}$$

where $Jf(x)$ denotes the Jacobian of the function $f(x)$. It is straightforward to re-cast Eq. 23 in the context of the current problem by replacing the vector function $f(x)$ by a relevant vector function for the present problem.

5.1 Domain Residuals

Newton's scheme is used to solve the entire system of coupled electro-mechanical problem together. The relevant vector functions used in the present case are called residuals.

Eq. (4) gives the electrical residual and Eq. (21) the mechanical. In addition, the auxiliary Eq. (5) is used in conjunction with Eq. (22) as an inter-domain coupling equation. It must be noted that the primary variables are B and $\mathbf{U} = [\mathbf{u}\ \mathbf{w}\ \theta]$ representing respectively the electrical and mechanical variables.

The electrical residual can be formed from Eq . (8), as

$$R_E(\mathbf{q},B) = \phi(\mathbf{X}^+) + \int_{S_1^+ \cup S_2^+} \frac{\log R(\mathbf{X}^+,\mathbf{Y}) B(\mathbf{Y})}{2\pi\varepsilon} dS(\mathbf{Y}). \tag{24}$$

Similarly the mechanical residual can be formed from Eq. (21):

$$R_M(\mathbf{q},B) = [\mathbf{M}]\{\ddot{q}\} + [\mathbf{K}]\{q\} - \{P(B,t)\} \tag{25}$$

5.2 Newmark Scheme for Time Integration

The Newmark method [18] is a widely used time integration scheme for dynamic analysis in finite element modeling. There are various ways of implementing Newmark scheme, one which is used in the present work is called the $a-$form [19]. Define *predictors*,

$$\begin{aligned} \tilde{\mathbf{q}}_{\mathbf{n+1}} &= \mathbf{q_n} + \Delta t \mathbf{v_n} + \frac{\Delta t^2}{2}(1-2\beta)\mathbf{a_n} \\ \tilde{\mathbf{v}}_{\mathbf{n+1}} &= \mathbf{v_n} + (1-\gamma)\Delta t \mathbf{a_n}. \end{aligned} \tag{26}$$

The next step is to use the *predictors* to obtain the actual quantities,

$$\begin{aligned} \mathbf{q_{n+1}} &= \tilde{\mathbf{q}}_{\mathbf{n+1}} + \beta \Delta t^2 \mathbf{a_n} \\ \mathbf{v_{n+1}} &= \tilde{\mathbf{v}}_{\mathbf{n+1}} + \gamma \Delta t \mathbf{a_{n+1}}. \end{aligned} \tag{27}$$

Here β and γ are algorithmic parameters that are fine tuned for integration accuracy and numerical stability. For a discussion on the effect of these parameters on the performance on the algorithm, see [19].

To start the process, $\mathbf{a_0} = \ddot{\mathbf{q}}(0)$ can be calculated from

$$\mathbf{M a_0} = -\mathbf{K q}(0) + \mathbf{P}(B(0)). \tag{28}$$

To march forward in time for acceleration, one needs to solve the time discrete version of the dynamic Eq.(21),

$$\mathbf{M a}_{n+1} + \mathbf{K q}_{n+1} = \mathbf{P}(B_{n+1}), \tag{29}$$

This equation set is nonlinear would be solved using the Newton scheme.

5.3 Implicit Time Integration

Finally, time integration for the problem is implemented using the Newmark scheme utilizing Newton's scheme. The dynamic equation can be formulated as,

$$\mathbf{M}\ddot{\mathbf{q}}(t) + \mathbf{K q}(t) = \mathbf{f}(B(t)). \tag{30}$$

Here $\mathbf{f}(B(t))$ denotes the entire force loading term obtained through BEM analysis.

Now define,

$$\mathbf{R}(\mathbf{q},B) = \begin{pmatrix} R_E \\ R_M \end{pmatrix} \tag{31}$$

Here, $\mathbf{R}$ is the grand residual for the problem. The Newton iterative scheme is essentially:

$$\begin{pmatrix} \frac{\partial R_E}{\partial B} & \frac{\partial R_E}{\partial \mathbf{q}} \\ \frac{\partial R_M}{\partial B} & \frac{\partial R_M}{\partial \mathbf{q}} \end{pmatrix}^{(k)} \cdot \begin{pmatrix} \Delta B \\ \Delta \mathbf{q} \end{pmatrix}^{(k)} = - \begin{pmatrix} R_E \\ R_M \end{pmatrix}^{(k)} \tag{32}$$

$$\mathbf{q}^{(k+1)} = \mathbf{q}^{(k)} + \Delta \mathbf{q}^{(k)} \quad B^{(k+1)} = B^{(k)} + \Delta B^{(k)} \tag{33}$$

We are using superscripts to denote Newton iteration step and subscript for Newmark integrator. Starting with $k = 0$, Eq. 32 is iterated until convergence. At convergence, $\mathbf{R}^{(k)} \equiv R(\mathbf{q}^{(\mathbf{k})}, B^{(k)}) \to \mathbf{0}$. This iteration helps us find the value of $\mathbf{a_n}$ needed at each step of time integration through an update of $\mathbf{q_n^{(k)}}$. The algorithm for the coupled scheme is described as below,

1. Solve BEM on ∂B for applied voltage and compute the traction $\mathbf{H^0}$ from Eq. (22).
2. Set initial values of displacement $\mathbf{q_0}$ and velocity $\mathbf{v_0}$ to $\mathbf{0}$ and compute initial acceleration using $\mathbf{a_0} = \mathbf{M^{-1} f^0}$ from Eq. (30)
3. Set $\mathbf{a}_{n+1}^{(0)} = \mathbf{a}_n, \mathbf{v}_{n+1}^{(0)} = \mathbf{v}_n$ and $\mathbf{q}_{n+1}^{(0)} = \mathbf{q}_n$.
4. Estimate $\tilde{\mathbf{q}}_{n+1}$ and $\tilde{\mathbf{v}}_{n+1}$ from $\mathbf{q}_n$ and $\mathbf{v}_n$ using Eq. (26).
5. $B_{n+1}^{(0)} = B_n$
6. Set $k = 1$
7. Newton iteration for time step $n+1$:
   (a) Use Eq. (24) and (25) to compute the value of requisite residuals.$B = B_{n+1}^{(k)}, \mathbf{q} = \mathbf{q}_{n+1}^{(k)}$.
   (b) Compute required four residual gradient from the relevant equations where $B = B_{n+1}^{(k)}, \mathbf{q} = \mathbf{q}_{n+1}^{(k)}$.
   (c) Update acceleration as $\mathbf{a}_{n+1}^{(k)} = 1/\beta\Delta t^2(\mathbf{q}_{n+1}^{(k)} - \tilde{\mathbf{q}}_{n+1})$ and $\mathbf{v}_{n+1}^{(k)} = \tilde{\mathbf{v}}_{n+1} + \gamma\Delta t \mathbf{a}_{n+1}^{(k)}$
   (d) $R_M^{(k)} = R_M^{(k)} + \mathbf{M a}_{n+1}^{(k)}$ and $\partial R_M/\partial \mathbf{q}|^{(k)} = \partial R_M/\partial \mathbf{q}|^{(k)} + 1/(\beta^2\Delta t^2)\mathbf{M}$
   (e) Plug the above residuals to Eq. (32) and solve for the increments.
   (f) Use Eq. (33) to update variables through increments.
   (g) Compute the tolerance.
   (h) Update k=k+1
   (i) If tolerance is high, repeat from step (7).
8. $\mathbf{a}_{n+1} = \mathbf{a}_{n+1}^{(k)}, \mathbf{v}_{n+1} = \mathbf{v}_{n+1}^{(k)}, \mathbf{q}_{n+1} = \mathbf{q}_{n+1}^{(k)}$
9. $B_{n+1} = B_{n+1}^{(k)}$
10. $n = n+1$ and repeat from step (3) till required time limit is reached.

6 Numerical Results

The method described above can be used to accurately simulate MEMS. Some of the mesh converged numerical results are discussed here. The time displacement curve for the MEMS under DC bias is shown in Fig .(3). The curve gives an excellent agreement with theoretical natural frequency of vibration.

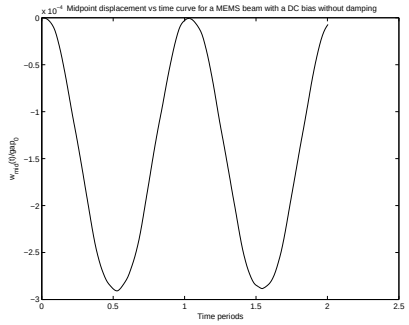

Figure 3: Time-Displacement curve for undamped MEMS. Time axis is nondimensionalized by natural time period of beam and displacement axis by initial gap

Fig . (6) shows an important MEMS phenomenon of pull-in instability [2]. According to Nayfeh et. al. [20], the pull in shall commence when the gap is approximately reduced to 43% of the original. The result from this analysis predicts the same results.

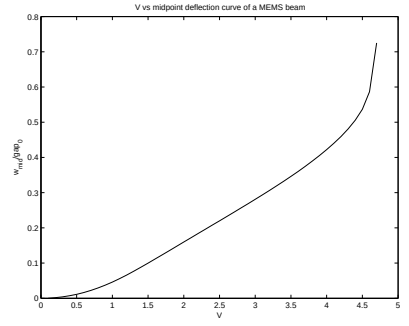

Figure 4: Voltage vs displacement curve for undamped MEMS. Static pull-in phenomenon

One of the interesting plots is Fig. (5) which shows $V^2 \propto F_{applied}$ vs displacement curve. The nature of the curve clearly shows the effects of competing nonlinearities in the entire process. The electrical effects tend to soften the beam whereas the mechanical membrane effect tends to stiffen the beam. The result of the curve clearly show that the final nature of the beam in this model is hardening indicating the mechanical stiffness to be of overpowering consequence.

If on the other hand, the MEMS beam is excited using an AC bias, we clearly see the resonance phenomenon. The model correctly predicts the fold-over characteristic of

[2]Pullin is essential for using MEMS as a bistable switch

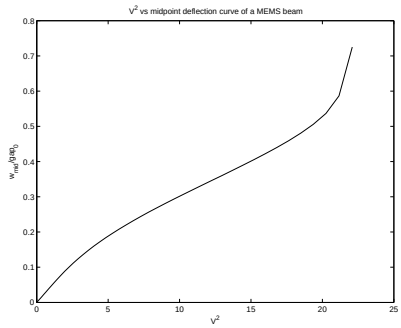

Figure 5: $V^2 \sim$ Force vs displacement curve. Competing nonlinearity of electrical and mechanical domain.

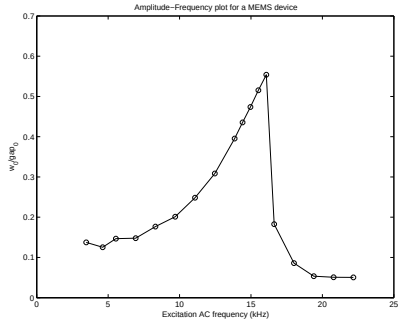

Figure 6: Amplitude vs frequency curve for undamped MEMS. Notice the nonlinearity of the beam near resonance

the resonance curve approximately around the natural frequency. The result shows excellent agreement with the analytical predictions.

7 Looking Ahead: Fast Multipole Accelerated BEM

One of the most important concerns of any simulation method is speed along with accuracy. A typical microdevice may have a huge number of MEMS packed in a small space. Computational speed is essential if system needs to be numerically analyzed in many different situations. The electrostatic field and stokes damping may need boundary element analysis which are typically slower and can prove bottlenecks in analysis. The other alternative of using finite elements runs into trouble because of the need for meshing such complicated configurations especially in full blown 3-D problems. The computational requirements for boundary element method are demanding and will require several generations of Moore's law to catch up with the inefficiencies. One of the most exciting developments in this area has been the fast multipole methods (FMM) developed first by Rokhlin [21] and later popularized by Greengard [22]. Explanation of the details of FMM is beyond the scope of the present paper and the reader is requested to read Nishimura [23] for an exhaustive review on this subject. The present

challenges are related to utilizing FMM for accelerating MEMS specific problems. Densely packed large number of Thin and ultra-thin structures present the next generation of challenges [6],[7]. The speed of computation promises to remain an important issue for any attempt at large scale CAD development.

8 Conclusions

A variety of topics relevant to MEMS simulation have been discussed and thin beam approximation method has been described in detail. The simulations show excellent agreement with theoretical data whenever available and have the promise to be integrated into a larger CAD substrate. In addition the method outline is extremely versatile and can be easily used to add layers of multiphysics and multidomain influences (see Appendix). With newer and more ambitious application possibilities, this market is set to scale new heights and will prove decisive to bring down the overall cost of design and fabrication.

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Appendix

A Damped Vibration Analysis Assuming Stokes Flow

In terms of Green's function solution, we can transform Eq. (1) in the following form,

$$\mathbf{v}(\mathbf{x}) = \int_{\partial B} \mathbf{T}(\mathbf{x},\mathbf{y})\mathbf{v}(\mathbf{y})ds(\mathbf{y}) + \int_{\partial B} \mathbf{G}(\mathbf{x},\mathbf{y})\tau(\mathbf{y})ds(\mathbf{y}), \quad \mathbf{x} \in \partial B \tag{34}$$

where $\mathbf{v}$ is the velocity of the fluid, $\mathbf{G}(\mathbf{x},\mathbf{y})$ the Green's function,$\mathbf{T}$ is the traction kernel, τ the traction due to fluid.

A.1 Forming Fluid Residual

Let us denote the electrical, mechanical and fluidic variables as E, M and F. It is now straightforward to construct the residual from Eq. (34),

$$R_F(E,M,F) = \mathbf{v}(\mathbf{x}) - \int_{\partial B} \mathbf{T}(\mathbf{x},\mathbf{y})\mathbf{v}(\mathbf{y})ds(\mathbf{y}) - \int_{\partial B} \mathbf{G}(\mathbf{x},\mathbf{y})\tau(\mathbf{y})ds(\mathbf{y}) \tag{35}$$

Analogous to the BIE electrostatic problem [5],[15] consider the flow in a region outside of, in this case, a single thin beam. (One beam is considered for simplicity of explanation - flow around many beams can also be easily modeled). It has been shown by Mukherjee et. al. [16] that for a thin beam, with $\mathbf{x}^+ \in s^+$,

$$v_i(\mathbf{x}^+) = g_i(\mathbf{x}^+) = \neq\!\!\!\!\int_{s^+} T_{ij}(\mathbf{x}^+,\mathbf{y})w_j(\mathbf{y})ds(\mathbf{y}) + \int_{s^+} G_{ij}(\mathbf{x}^+,\mathbf{y})q_j(\mathbf{y})ds(\mathbf{y}), \quad \mathbf{x}^+ \in s^+ \tag{36}$$

where $q_j = \tau_j^+ + \tau_j^-$ and $w_j = v_j^+ - v_j^-$.

For a thin beam $v_j^+ \approx v_j^-$, causing the first integral on the right hand side to disappear in Eq. (36). The above equations then simplifies to:

$$v_i(\mathbf{x}^+) = g_i(\mathbf{x}^+) = \int_{s^+} G_{ij}(\mathbf{x}^+,\mathbf{y})q_j(\mathbf{y})ds(\mathbf{y}), \qquad \mathbf{x}^+ \in s^+ \tag{37}$$

It has been shown by Mukherjee et.al. [16] that null space of the kernel G is empty and Eq. (37) has a unique solution for any prescribed velocity $\mathbf{g}(\mathbf{x})$ on $\partial B = s^+ \cup s^-$.

A.2 Newton's Method Implementation

Once the residuals and gradients are computed, the Newton's method can be implemented as before,

$$\begin{pmatrix} R_{EE} & R_{EM} & R_{EF} \\ R_{ME} & R_{MM} & R_{MF} \\ R_{FE} & R_{FM} & R_{FF} \end{pmatrix}_n \cdot \begin{pmatrix} \Delta E \\ \Delta M \\ \Delta F \end{pmatrix}_n = \begin{pmatrix} R_E \\ R_M \\ R_F \end{pmatrix}_n$$

where R_{AB} denotes $\partial R_A/\partial B$, and $E_{n+1} = E_n + \Delta E_n$, $M_{n+1} = M_n + \Delta M_n$, $F_{n+1} = F_n + \Delta F_n$.

The fluid variable for our case would be fluid traction $\{\tau\}$ which will contribute to the right hand side of Eq. (30).