Lazare Carnot's theory of machines and the background of Ampère's introduction of the word kinematics.

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Abstract

Circa 1830 Ampère developed a new classification of human knowledge, which included the definition of a new sub-science of mechanics called *kinematics*. In this paper I will discuss the background of this proposal. I will in particular discuss Lazare Carnot's *Essai sur la composition des machines* of 1783 and his *Principes fondamentaux de l'équilibre et du movement* of 1803. Lazare Carnot started the investigation of machines in terms of energy. Moreover Carnot's approach is such that it will become clear as well that an independent geometrical science of motion that does not take masses and forces into consideration was in the air.

Keywords: Lazare Carnot, Ampère, kinematics, history of mechanism and machine science,

1. Introduction

Mechanical engineering is essentially a multidisciplinary activity (Cf. [1]). Unlike, for example, mathematics, physics or chemistry, mechanical engineering primarily deals with artifacts, entities that are man-made. Moreover, because machines are functioning in the real world the many different aspects that they possess are in principle all important. Machines possess economic, legal and other cultural aspects as well. New inventions and new technologies can lead to changes in existing machines or to the introduction of new machines. This obviously leads to situations in which mechanical engineers must adapt to the new circumstances and this sometimes involves the learning of other disciplines. The notion 'the best possible machine' is in principle not only dependent on new developments in the sciences but in addition on, for example, the introduction of new laws or economic developments. As long as there are no environmental laws the designer of an engine can ignore the nature of the gasses that the engine releases in nature. New legislation can change this drastically. The laws of nature are independent of human institutions but yesterday's best possible machine can easily cease to be the best possible machine today because of external developments. This multidisciplinary character of mechanical engineering science (MMS) is reflected in its history. In the development of MMS first the *aspect of machines that can be handled by means of statics* was studied. The functioning of the basic machines like the lever, the pulleys, the wedge and the screw was explained by means of the law of the balance: there exists equilibrium if the weights are inversely proportional to the distances from the center. Big machines were viewed as combinations of simple machines and understanding the simple machines was viewed as implying the understanding of the combinations as well.

In particular during the industrial revolution the complexity of the new machines led to the insight that the great variety in the geometry of machines requires separate investigation. Machines came to be seen as compounded of mechanisms and mechanisms could be classified according to the way in which they transform motion. This led to Monge's classification of mechanisms, first published by Lanz and Betancourt [2]. About 1830 Ampère coined the word 'kinematics' for a new sub-science of mechanics which would deal with motion without taking forces and masses into consideration. The *geometrical or kinematical aspect of machines* and mechanisms was studied extensively in the 19th century.

However, other aspects of machines were theoretically studied as well. In 1783 Lazare Carnot initiated the investigation of machines in terms of energy with the publication of his *Essai sur les machines en general*. This *energetic aspect of machines* was also studied by Coriolis in 1829 ([3]). Cf. also [6]. Lazare Carnot's son, Sadi Carnot, and the other authors on thermodynamics can be viewed as dealing with this aspect of machines.

Below I will discuss Lazare Carnot's approach to machines. We will see how he deals with the energetic aspect of machines. In his considerations Carnot introduced the notion of 'geometrical movement', which he considered as so important that in 1803 he even wrote about a "special science of geometrical movements", that had to be developed. It was no accident that Ampère when he introduced the word 'kinematics' referred to both Lazare Carnot and to Lanz & Betancourt ([9], p. 48).

2. Biography of Lazare Carnot

Lazare Carnot (1753-1823)[1] was trained at the military school in Mézières where the abbé Charles Bossut was professor. Bossut was, moreover, the author of a successful textbook, *Traite élémentaire de mécanique et de dinamique appliqué principalement aux mouvemens des machines*, Charleville, 1763 that was used at the school. So it may have been Bossut's book that triggered Carnot's interest in machines. The second 'book' of Bossut's 1763 textbook on mechanics, applied in particular to machines, is devoted to the communication of movements, that is, the theory of collisions. This theory with respect to collisions of hard (non-elastic) bodies was applied by Carnot. However, it is very probable that Carnot's main source of inspiration was D'Alembert's *Traité de Dynamique* (1758).

Before Carnot, the theory of machines was usually restricted to the elementary machines, of which the functioning was explained by means of the laws of statics. Beyond such considerations there was no coherent theory of machines at the time. This is understandable. The rational mechanics of the 18th century itself was very much under construction. A principle like potential energy plus kinetic energy is constant (in a conservative force field), familiar to every student of mechanics, was unknown. The notion of 'work' did not exist either. A famous 18th century issue concerned the question whether momentum = mass times velocity or 'life force' = mass times velocity squared was the fundamental quantity conserved when motion was transferred. In the collision theory of hard and elastic bodies momentum was conserved. In collision theory of perfectly elastic bodies the live force was conserved as well, but not in the collision theory of 'hard bodies'. The notion 'hard body' is important on 18th century mechanics (Cf. [8]). The ultimate corpuscules in nature were imagined to be completely impenetrable. Hard solid matter was considered to consist in such corpuscules connected to each other by perfectly rigid rods. In elastic matter the rods were imagined as springs.

It seems that Carnot as a young lieutenant in the Royal Corps of Engineers, after having graduated on January 1, 1773, remained interested in mechanics and studied works by D'Alembert and Euler. Garrison duty at Calais, Le Havre and elsewhere must have been pretty boring for an ambitious and intelligent young man.

Then, on April 18, 1777 the *Gazette de France* announced a prize contest organized by the Academy of Sciences in Paris. The Academy requested for the 1779 prize a treatise about "the theory of simple machines with regard to friction and the stiffness of cordage". It was required that the laws of friction and the investigation of the effects of the stiffness of cordage be determined by means of new experiments conducted on large scale. It was further required that these experiments would be applicable to machines used in the Navy such as the pulley, the capstan, and the inclined plane". ([7], p. 272). Carnot read the announcement and decided to try his luck. His treatise, written in less than a year, was received by the Academy on March 28, 1778. Unfortunately the judges felt that Carnot had not answered their questions satisfactorily. Actually, they found none of the submitted papers good enough. That is why the Academy set the same problem again in 1781. Carnot gave it another try but this time C. A. Coulomb won, although Carnot was awarded honorable mention. Coulomb's paper, « Théorie des machines simples en ayant égard au frottement et à la roideur des cordages » (*Mémoires des Savants étrangers*, Vol X), is nowadays considered as one of the first important contributions to the theory of friction. It is a model of experimental analysis in which the friction of oak on oak, oak on fir, fir on fir, etc. are systematically studied.

It is not surprising that Carnot's treatise was rejected by the Academy. The Academy clearly wanted experiments and results applicable in the Navy. Carnot did some experiments, but the title of his treatise was not accidentally *Mémoire sur la théorie des machines*. In the treatise Carnot started to develop a general theory of machines. In the next few years he turned his treatise for the Academy into a book which appeared in 1783 under the title *Essai sur les machines en general* (Defay, Dijon,1783).

The French Revolution started in 1789. In June 1793, a critical phase in the history of France, when Austria and Prussia had both declared war on France with the aim to restore the rule of King Louis XVI , Carnot became member of the Comité du Salut Public responsible for military affairs. For four years Carnot shared in the supreme power in France, mainly dealing with military matters. Carnot was a child of the age of Enlightenment and a firm believer in the value of science and technology. He got involved with the mathematician Gaspard Monge, the chemists Jean-Antoine Chaptal (1756-1832)[2], Claude Louis Berthollet (1748-1822) and others with the aim to integrate French science and technology in the defense of the country. Monge rewrote a handbook on the production of guns, Chaptal was from 1974 in charge of the production of gunpowder. A part of this

[1] Charles Coulston Gillespie, *Lazare Carnot Savant*, Princeton University Press, Princeton, New Jersey, 1971

[2] Chaptal was one of the first chemists to accept Lavoisier's ideas. He is important in the history of economic thought as well: Elsa Bolado and Luis Argemi, Jean Antoine Chaptal: from chemistry to political economy , *The European Journal of the History of Economic Thought*, 12:2, pp. 215-239

process was also the creation in 1794 of the *École centrale des travaux publics* that got the name *École Polytechnique* one year later in 1795.

The Academy was abolished in 1793 and in 1795 replaced by the Institute. In 1796 Carnot became a member. After 1800 the Institute became Carnot's chief interest, serving on many commissions dealing with technology. At the request of others, as he wrote himself, in 1803, Carnot published an elaborated version of his 1783 book with the title *Principes fondamentaux de l'équilibre et du movement* ([5]). The book does not contain anything new compared to the early version, but the second version makes the understanding of the early very concise version much easier.
Carnot supported Napoleon all the way, also after Napoleon's return from Elba. In the 'government of the hundred days' Carnot was minister of the interior. When Napoleon was imprisoned on St. Helena, Carnot had to go into exile. He died in 1823 in Magdeburg.

3. Carnot's theory of machines 1

Let me try to reduce Carnot's theory of machines to a few central ideas.

i) The mass of the parts of a machine must be taken into consideration. One can study a lever or a system of pulleys by abstracting from the mass of the mechanism and by studying the equilibrium of a few forces. However, in a general theory of machines the mass of all parts of the machine must be taken into consideration.
ii) There is another reason why Carnot rejects the classical approach based on the laws of statics to the elementary machines. The notion of force is for Carnot a metaphysical notion unless it is defined in terms of empirical notions like mass and velocity. A theory of machines should be based upon only a theory of the communication of movements. What Carnot in fact said was: kinematics first, then dynamics, and statics is a special case of dynamics.
iii) A machine is a connected system of (hard) bodies. The connections between the bodies constrain the movement of the bodies. The geometry of the system determines which motion is possible and which is not.
iv) The constraints mentioned in iii) cause interactions between the bodies.
v) These interactions can take place smoothly (in Carnot's words: "par degrees insensibles") or by means of collisions. Because such smooth interaction can be viewed as the result of a sequence of infinitesimally small percussions, we need only one theory for both cases. This theory is based on a theory of collisions.

Imagine a machine consisting of hard bodies. The first thing to study is the interaction at a certain moment in time of two of such hard bodies. That is what Carnot did.
Imagine we have a body of mass M, which possesses before the interaction velocity W and after the interaction velocity V. Then we have a body of mass M', which has before the interaction velocity W' and after velocity V'. Nota bene: Carnot treats velocities in fact as vectors although he does not possess the full notational apparatus of the vector calculus. This makes the reading of Carnot somewhat difficult. It depends on the context how U, V and W must be interpreted. By definition U=W-V and U'=W'-V' (these are vector equations) are the velocities lost by the two masses in the collision. Z is the angle between respectively U and V; Z' is the angle between U' and V'. The interaction of the bodies can be understood in terms of the fundamental laws of the theory of collisions: i) Action is reaction directed along the normal to the touching surfaces (action and reaction are forces determined by multiplying a mass with a velocity), ii) After the shock the relative velocity of two touching surfaces in the direction of the normal is zero. Hard bodies do not bounce. The idea to treat a system of interacting bodies in this way comes from D'Alembert's *Traité de Dynamique* (Cf [10], pp. 248-253).

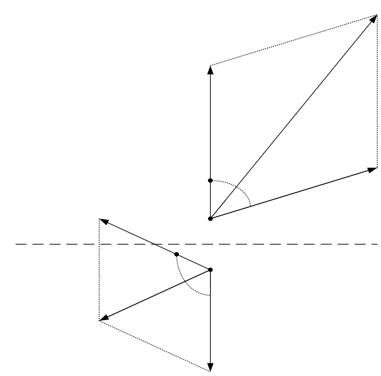

Figure 1

This leads to the following two equations:

$$MU=M'U',$$

$$V\cos Z=-V'\cos Z'.$$

Nota bene: In these equations U, U', V' and V are scalars and no longer vectors. The first one expresses: Action is reaction. The second one: After the interaction the bodies have equal relative velocities These two equations yield by multiplying them[3] the following scalar equation:

$$MVU\cos Z + M'V'U'\cos Z' = 0.$$

In the *Essai* Carnot immediately generalizes to a multi-body system and he writes his *First Fundamental Equation*

$$\sum mVU \cos <U,V> = 0. \qquad \text{(E)}$$

N. B. In ‘$\cos <U,V>$’ where $<V,U>$ is the angle between V and U, U and V are vectors; in ‘mVU’ they are scalars. This generalization (E) to a multi-body system is not a completely trivial move. In the *Principes* Carnot carefully considered the case that a hard body interacts with several other hard bodies at the same time. Formula (E) still holds.

Formula (E) is powerful. For example, we have for all vectors W, V, U with W=V+U, the scalar equation:

$$W^2=V^2+U^2+2VU \cos<V,U>,$$

which implies (Cf. *Principes* p. 146) with (E):

$$\sum MW^2=\sum MV^2+\sum MU^2.$$

Figure 2 Lazare Carnot. Lithograph (22x16 cm) , Print artist: Ducarme, 19th C. Original Artist: R. J.
Courtesy of the Dibner Library portrait collection.

On the basis of this scalar formula Carnot draws on p. 148 of the *Principes* the conclusion that if we avoid collisions and the motion in a machine only changes smoothly, “par degrees insensibles”, ΣMU^2 is so small (of the second order, he says) that we can ignore it and we get

$$\sum MW^2=\sum MV^2.$$

On p. 255 of the *Principes* Carnot argued that percussions or other sudden changes in the functioning of a machine should be avoided. They cause the loss of live force, “déperdition de forces vives”. J. A. Borgnis wrote in 1823 in his *Dictionaire de mécanique appliqué aux arts* (pp. 57-58) that Carnot had advocated to boost the efficiency of machines by avoiding to-and-fro motions and eliminating sudden impacts (according to [8], p. 204)

The geometry of the constraints of a system of hard bodies determines which motions are possible and which not. An important question in a theory of machines is undoubtedly the question which of the possible motions is the one that actually takes place.

In this respect one of the notions that Carnot introduced and of which he was proud was the notion of *geometric movement* (mouvement géométrique). In the *Essai* Carnot considered the reversibility of a movement as characteristic of this type of movement (pp. 23-24). In the *Principes* he defines a geometric movement as movements that do not have any effect on the interaction of the bodies on each other. In the *Principes* he proves from the definition that the inverse of a geometric movement is another geometric movement. He also proves that the combination of two geometric movements is another geometric movement.
Let us see how Carnot derives his second fundamental law. Consider the first one:

$$\sum mVU \cos <U,V> = 0. \qquad \text{(E)}$$

In this formula U=W-V is the velocity distribution lost when the interaction turned an initial velocity distribution W into a velocity distribution V. The velocity distribution V is obviously an example of a geometric movement. Let u be an arbitrary geometric movement. Then u-V is a geometric movement as well (the inverse –V of V is one and the sum of u and –V is one). Because a geometric movement does not interfere with the interaction of the bodies, we can simply superpose it on the system before an interaction. Then we get an initial velocity distribution W+(u-V). The resulting velocity distribution after interaction will be V+(u-V)=u. The velocity distribution that is lost will be W+(u-V) – (V+(u-V))=W-V=U. So application of (E) yields the *Second Fundamental Formula*:

$$\sum muU \cos <U,u> = 0 \qquad \text{(F)}$$

[3] In the *Essai* the derivation is on pp. 15-22 and in the *Principes* on pp. 131-143

for an arbitrary geometric movement u and all possible distributions U of velocities that can be lost because of interaction.

(F) is a powerful formula as well. Carnot proves that after sudden change in the state of a system the geometrical movement that actually results is the geometrical movement that corresponds to a minimum of $\sum MU^2$.

Proof: We have (F). Suppose that instead of the movement V actually occurring after the sudden change the movement V-u' (u' is an infinitesimal geometrical movement) occurs. We do not change W, the movement before the sudden interaction of the hard bodies. The new velocity lost is then, obviously, U' =W-(V-u') = V +U-V + u' = U+u'. So u' is the vector U increases as a result of the change in V. then we have from (F):

$$\sum mu'U \cos \langle U,u' \rangle = 0 \text{ or}$$

$$\sum mU\, u'\cos \langle U,u' \rangle = 0,$$

but then, u'cos<U,u'> is the scalar increment of U (the projection of u' on U). We can call it δU, the increase in the length of U. We get:

$$\sum mU\, \delta U = 0 \text{ or } \delta \int mU^2=0.$$

Supposed infinitesimal changes in the output velocity after the sudden interaction of the bodies show that $\int mU^2$ must possess an extremum.

4. Carnot's theory of machines 2

Let us now look at some of Carnot's conclusions with respect to machines.

A crucial notion is the notion of kinetic energy. As we have seen Carnot uses the term "living force "(force vive") and uses the formula mv^2 for it, instead of the modern ½ mv^2. However, he also has the notion "latent living force". A weight P with mass M at a height of H has a latent living force (we would nowadays say potential energy) equal to P.H. When the weight falls, after having covered the distance H, its velocity is V and its living force equals ½ mV^2.The latent living force has been turned into actual living force: PH=½ MV^2.

It is interesting that Carnot describes an animal as an assembly of springs that contains latent kinetic energy which can be turned into actual kinetic energy.

Another important notion is "momentum of activity" or "effect" (We would nowadays say work). A weight P with mass M can be lifted with uniform velocity V in time T over a distance H with a constant lifting force F. Then the moment of activity or effect equals FMT. And FMT=PH. Here we see that that a "momentum of activity" can be turned into latent living force, which can be turned into living force by dropping the weight again.

Clearly Carnot is very close to the notion of work in relation with the principle of the conservation of mechanical energy.

As for machines Carnot points out that in machines it is the "moment d'activité" that one must "economize as much as possible". Losses of "moment d'activité" are to be avoided in machines. Unnecessary motion should be avoided. For example, water arriving in a reservoir with a velocity higher than necessary will cause unnecessary consumption of "l'effort de la puissance motrice". And finally friction, air resistance and the like ought to be avoided.

5. Carnot: the science of geometric movements

In the preface of the *Principes* Carnot argues that there are two ways to consider the principles of mechanics. The first one is to define mechanics as the theory of forces. The second one is to define mechanics as the theory of motion. Clearly Carnot prefers the second definition. In the first case one starts with an obscure notion, the notion of force. However, if one accepts this notion one can postulate the axioms of statics, develop statics and relate motion to force as follows: One introduces a fictitious force equal in magnitude to the product of the mass of the body and its acceleration directed opposite to the acceleration. This leads to a condition of kinetic equilibrium. Although this first approach was generally accepted at the time Carnot chose the second approach. In this context he introduced his theory of geometric movements. Consider the following quotation:

> "*The theory of geometrical movements is very important; it is […] a kind of science intermediate between ordinary geometry and mechanics. It is the theory of the movements that an arbitrary system of bodies can make in such a way that they do not hinder each other or exert some action or reaction on each other.* This science has never been dealt with in particular. *This science must be created completely and it deserves for its beauty and its utility the full attention of the scholars*; because the great analytical difficulties one meets in mechanics and especially in hydraulics are only caused by the fact that a theory of geometrical movements has not been created at all." [*Principes*, p. 116; italics are mine].

These geometrical movements are completely determined by the geometry of the machine. On an abstract level this view of machines is related to Monge's view of machines elaborated in the *Essay on the composition of machines* prepared by Lanz and Betancourt [2] in which the emphasis is also very much on the geometry of the machines.

6. Ampère

Ampère coined the word 'kinematics'. When he did so, not only the views of Carnot and Monge on machines were on his mind. Maybe the fact that he had decided to give a new classification of the sciences was even more important. I will describe the background of this new classification and the position of kinematics in it. The classification very much reflects the state of science in the first half of the 19th century.

In 1829 when A.– M. Ampère was preparing a course on general and experimental physics for the Collége de France, he felt an understandable need to define and subdivide the subject of the course. While doing so he realized that the time had come to answer questions that he had been asking himself for a long time on human intellect, on its development, on the way the true and the false ought to be distinguished, on the methods that must be used to classify the objects of our knowledge or our knowledge itself. He decided to answer two questions:

1. What is general physics and by which precise characteristic can it be distinguished from the other sciences?
2. Which are the different branches of general physics?

The answer to the first question was: general physics deals with the inorganic properties of bodies in so far as they are independent of their use (that is a subject for technology) and independent of time, place and climate (this separates physics from physical geography).

As for the second question Ampère distinguishes two different points of view from which one can study the object of general physics: the first point of view encompasses everything that observation and experience can let us know about the inorganic properties of the bodies themselves (*general elementary physics*). The second point of view (*mathematical physics*) concerns the general laws that result from the comparison of what we observe, the causes and the consequences that we deduce from the laws. These two points of view are each once more subdivided in two subordinated points of view. Some of the properties of bodies we can observe immediately; others concern what is hidden in the bodies, the elements they are made of. So *general elementary physics* consists of *experimental physics* and *chemistry*. In chemistry the properties of the elements cannot be immediately observed; we need analysis. *Mathematical physics* is subdivided as well in *stereonomy* (stéréonomie), which deals with the comparative study of the means that are used to make experiments as precise as possible and the formulae that result from the experiments, and the part of mathematical physics that deals with the research concerning causes and the laws.

Ampère, André-Marie (1775 - 1836)
Artist graphic : Ambroise Tardieu, 1788-1841
Courtesy of the Dibner Library portrait collection.

Ampère discovered soon that a similar subdivision could be applied to other sciences. In the spring of 1830 he succeeded to do this for all *cosmological sciences*, which encompass mathematics, physics and others. Later he saw that he could apply a similar principle to the sciences that he called *noological sciences*, the sciences dealing with human thought and human societies. The basic idea is the following: We divide the knowledge of everything that exists in *two* realms (règnes) on the basis of the characteristics of the object that is being studied: cosmological sciences and noological sciences.

Each of these two is subdivided in *four* branches and the four branches are each subdivided in *sixteen* first order sciences. So we have altogether *sixty four* first order sciences. There are four mathematical first order sciences: arithmology, geometry, mechanics and uranology (=astronomy).

Once we have defined the object of a first order science we

i) observe it and we collect facts at the surface, and we
ii) find out which facts are hidden under these surface facts,

Moreover, we find out what brings about these facts by

iii) comparing the observations, classifying them and deducing laws from them, and by
iv) studying the causes and the effects

This leads in Ampère's proposal to four third order sciences in mechanics: Two branches of *Elementary mechanics*: *kinematics and statics;* Two branches of

Transcendental mechanics: dynamics and molecular mechanics.

Molecular mechanics refers to scattered attempts to apply the laws of dynamics to molecules and explain in this way properties of rigid bodies. By the way in Ampère's proposal Molecular geometry is a science of the third order in geometry dealing with crystallography.

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