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Isomorphism and Inversions of Kinematic Chains Up to 10-Links

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Abstract

This paper presents a new method for the identification of inversions or the distinct mechanisms (DM) from a given kinematic chain (KC). The method is based on Weighted Physical Connectivity Matrix [WPCM] of the given KC. The two structural invariants namely sum of absolute characteristic polynomial coefficients [WPCMP∑] and Maximum absolute value of characteristic polynomial coefficient [WPCMPmax] have been derived and used as the identification numbers of the kinematic chains. The basic aim of this work is to facilitate the designer at the conceptual stage of design to select the best mechanisms/kinematic chains for the required specific task.

Key Words: Kinematic Chain; Inversion; Kinematic Pair; Weighted Physical Connectivity Matrix; Characteristic Polynomials.

1 Introduction

Over the past several years, much work has been reported in the literature on the structural synthesis of kinematic chains and mechanisms. Undected isomorphism results in duplicate solution and unnecessary effort. Therefore, the need for a reliable and efficient algebraic method for this purpose is necessary. Identifying isomorphism among kinematic chains using characteristic polynomials of adjacency matrices of corresponding kinematic chains are simple methods [1-3]. But the reliability of these methods was in questions as several counter examples were found by Mruthyunjaya [4]. The test proposed by Mruthyunjaya [4] is based on characteristic coefficients of the 'Degree matrix' of the graph of the kinematic chains. The elements of the degree matrix were sum of the degree of vertices (degree or type of links) or unity in a link-link adjacency matrix. Later on, this test was also found unreliable. Krishnamurthy [5] proposed the representation polynomial for detecting isomorphism between two kinematic chains. The representation polynomial is the determinant of the generalized adjacency matrix, called representation matrix of the KC. But the representation matrix requires the use of a large number of symbols, the calculation and comparison of the representation polynomials is not as easy as that of the characteristic coefficients of the adjacency matrix. Balasubramanian and Parthasarthy [6] proposed the procedure based on the concept of the permanent of a matrix for the purpose. Tang and Liu [7] presented a method based on degree code as mechanism identifier. The concept that the methods based on either link-link adjacency or joint-joint adjacency hardly differ in nature and are likely to fail at some stage or the other, seems to be unjustified. Of course, the characteristic methods by Uicker and Raicu [1], Degree matrix proposed by Mruthyunjaya [4] failed. But several other methods [8] are in use. After reviewing the literature, it has been concluded that a number of techniques are available for the generation of kinematic chains, detecting isomorphism and their mechanisms identification. With regards to these methods, there is either a lack of uniqueness or difficult to grasp.

In the present paper, a new method to identify the inversions from given kinematic chains is proposed. A new [WPCM] matrix is defined. The two structural invariants [WPCMP∑] and [WPCMPmax] are derived from [WPCM] matrix. These structural invariants are calculated using Software MAT LAB and are same for identical or structural equivalent kinematic chains /mechanism but different for distinct mechanism KC.

2 Representation of Kinematic Pairs

Nomenclature
C: Cylinder lower pair
F: Planer lower pair
G: Spheric lower pair
HP: Higher pairs (point contact)
HL: Higher pairs (line contact)
P: Prismatic lower pairs
R: Revolute lower pairs
SL: Screw lower pairs

The two links are connected either by lower pair or by higher pair. To distinguish them in physical connectivity matrix [PCM], the lower pairs and higher pairs are represented by numeric '1' and '2' respectively. Lower pairs and higher pairs are further distinguished based on the respective pairs with the number followed by decimal. All the kinematic pairs (KP) are distinguished by assigning different numeric values. Let R=1.1, P=1.2, C=1.3, SL=1.4, F=1.5, G=1.6, HP=2.1 and HL=2.2. These values are assumed to distinguish the KP.

2.1 Physical Connectivity Matrix [PCM]

Once the links of the mechanism have been numbered from 1 to n, [PCM] is defined as a square symmetric matrix of order n. The elements of [PCM] are entered with either zero or the type of kinematic pair and defined as:

$$[PCM] = \{P_{ij}\}_{nxn}, \quad (1)$$

Where, P_{ij} = Type of kinematic pair between i^{th} link and j^{th} link that are directly connected.

= 0, when i^{th} and j^{th} links are not connected directly.

Off course; P_{ii} =0

The form of [PCM] matrix will be:

$$[PCM]= \begin{pmatrix} 0 & P_{12} & P_{13} & - & - & - & P_{1n} \\ P_{21} & 0 & P_{23} & - & - & - & P_{2n} \\ - & - & - & - & - & - & - \\ - & - & - & - & - & - & - \\ P_{n1} & P_{n2} & P_{n3} & - & - & - & 0 \end{pmatrix}$$

2.2 Degree Vector (V)

The degree of link actually represents the type of link like binary, ternary, quaternary etc. Let

$d(v_i)$ =2, for binary link, $d(v_i)$ =3, for ternary link,
$d(v_i)$ = 4, for quarter nary link, $d(v_i)$ =k, for k-nary link.

The degree vector (V) represents the degree of individual link and is defined as

$$V = [v_1\ v_2\ v_3\ v_4\ v_5 - - - - v_n] \quad (2)$$

2.3 Relative Weight of Degree of Vector

Critical study of kinematic structure has revealed that there is a strong correlation between the network formed by the different types of links, KP, and the machine performance like wear, reliability, and susceptibility to manufacturing error because of link tolerances and joint/bearing clearance etc. So, this information should also be added in the [PCM] matrix in the form of relative importance of v_i to v_j and vice versa to construct a [WPCM] matrix.

$$W_{ij} = ½\,[v_i/v_j+v_j/v_i] \quad (3)$$

Here, Vi and v_j are the type of i^{th} and j^{th} links connected directly.

3 Weighted Physical Connectivity [WPCM] Matrix

For increasing the discrimination power for detection of isomorphism and identification of DM, the mutual effect of W_{ij} are introduced into each element of [PCM] matrix and so [WPCM] is defined as

$$[WPCM]= \{g_{ij}\}_{nxn} \quad (4)$$

$g_{ij} =(P_{ij})$ x (W_{ij})

The form of [WPCM] matrix will be:

$$[WPCM]= \begin{pmatrix} 0 & g_{12} & g_{13} & - & - & - & g_{1n} \\ g_{21} & 0 & g_{23} & - & - & - & g_{1n} \\ - & - & - & - & - & - & - \\ - & - & - & - & - & - & - \\ g_{n1} & g_{n2} & g_{n3} & - & - & - & 0 \end{pmatrix}$$

3.1 Characteristic Polynomial of [WPCM] Matrix

The characteristic polynomial is generally derived from (0, 1) adjacency matrix. Many researchers have reported co-spectral graphs (the non-isomorphic KC having same Eigen spectrum derived from (0, 1) adjacency matrix). But the Proposed [WPCM] matrix has additional information about the types of KP existing in a mechanism KC and it is verified that the characteristic polynomial and the coefficients of [WPCM] matrix are unique to clearly identify the mechanisms and even KC with co-spectral graphs.

The characteristic polynomial of degree n is given as:

$$|(WPCM - \lambda I)| = \lambda^n + a_1\lambda^{n-1} + a_2\lambda^{n-2} +\text{--------}+ a_{n-1}\lambda + a_n$$

Where;

1, a_1, a_2, a_{n-1}, a_n are the characteristic polynomial coefficients.

The two important properties of the characteristic polynomials are

(**a**) [WPCMP∑] is an invariant for a [WPCM] matrix. i.e.

$|1| + |a_1| + |a_2| + \text{-----} + |a_{n-1}| + |a_n|$ = [WPCMP∑]

(**b**) [WPCMPmax] is another invariant for a [WPCM] matrix.

3.2 Isomorphism of Kinematic Chains

Theorem: Two similar square symmetric matrices have the same characteristic polynomial [9]

Proof:

Let the two KC are represented by the two similar matrices A and B such that $B = P^{-1}AP$, taking into account that the matrix λI commutes with the matrix P and $|P^{-1}| = |P|^{-1}$. Since the determinant of the product of two square matrices equals the product of their determinants, we have

$$|B - \lambda I| = |P^{-1}AP - \lambda I|$$
$$= |P^{-1}(A - \lambda I)P|$$
$$= |P^{-1}|\,|(A - \lambda I)|\,|P| = |A - \lambda I|$$

Hence, D (λ) of 'A' matrix = D (λ) of 'B' matrix.

D (λ) =characteristic polynomial of the matrix.

It means that if D (λ) of two [WPCM] matrices representing two KC is same, their structural invariants 'WPCMP∑' and 'WPCMPmax' will also be same and the two KC are isomorphic otherwise non-isomorphic chains.

4 Structural Invariants [WPCMP∑] and [WPCMPmax]

The characteristic polynomial is generally derived from (0, 1) adjacency matrix. Many researchers have reported co-spectral graphs (the non-isomorphic KC having same Eigen spectrum derived from (0, 1) adjacency matrix). But the Proposed [WPCM] matrix has additional information about the types of KP and types of links existing in a mechanism KC. It has been verified that the characteristic polynomial and characteristic polynomial coefficients of [WPCM] matrix are unique to clearly identify the mechanisms and even KC with co-spectral graphs. The proposed [WPCM] matrix provides distinct set of characteristic polynomial coefficients of the kinematic chain with co-spectral graph also. Here the new composite invariants

[WPCMP∑], [WPCMPmax] of [WPCM] matrix are proposed and are obtained by using software MATLAB.

5 Tests for Distinct Mechanisms

The mechanisms are obtained by fixing the links of the KC turn in turn. If link 1 is fixed, the diagonal element P_{11} of [WPCM] is changed from 0 to 1. Then it will be the representation of the first mechanism with link 1 fixed. The structural invariants of [WPCM] matrix of the first mechanism are then calculated. These invariants are the characteristic numbers of the first mechanism. This process is repeated for the second link and so on. In this way, sets of invariants equal to the number of the links are obtained. Some of them are same and others are different. The same structural invariants represent the corresponding structurally equivalent links that constitute one DM.

6 Example-1

Considering 2 KCs with 10- bars of class IV (h) as shown in Fig 1 and 2 in which the binary (n_2) and 5-nary (n_5) links are same. WA and WB represent the [WPCM] matrices of chain 1 and 2 respectively.

$$WA=\begin{pmatrix}
0 & 1.1 & 0 & 1.595 & 0 & 1.595 & 0 & 1.595 & 0 & 1.595 \\
1.1 & 0 & 1.595 & 0 & 1.595 & 0 & 1.595 & 0 & 1.595 & 0 \\
0 & 1.595 & 0 & 1.1 & 0 & 0 & 0 & 0 & 0 & 0 \\
1.595 & 0 & 1.1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1.595 & 0 & 0 & 0 & 1.1 & 0 & 0 & 0 & 0 \\
1.595 & 0 & 0 & 0 & 1.1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1.595 & 0 & 0 & 0 & 0 & 0 & 1.1 & 0 & 0 \\
1.595 & 0 & 0 & 0 & 0 & 0 & 1.1 & 0 & 0 & 0 \\
0 & 1.595 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.1 \\
1.595 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1. & 0
\end{pmatrix}$$

$$WB=\begin{pmatrix}
0 & 0 & 0 & 1.595 & 0 & 1.595 & 0 & 1.595 & 1.595 & 1.595 \\
0 & 0 & 1.595 & 0 & 1.595 & 0 & 1.595 & 0 & 1.595 & 1.595 \\
0 & 1.595 & 0 & 1.1 & 0 & 0 & 0 & 0 & 0 & 0 \\
1.595 & 0 & 1.1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1.595 & 0 & 0 & 0 & 1.1 & 0 & 0 & 0 & 0 \\
1.595 & 0 & 0 & 0 & 1.1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1.595 & 0 & 0 & 0 & 0 & 0 & 1.1 & 0 & 0 \\
1.595 & 0 & 0 & 0 & 0 & 0 & 1.\ 1 & 0 & 0 & 0 \\
1.595 & 1.595 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1.595 & 1.595 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}$$

Structural Invariants of Kinematic Chains

The Structural Invariants (identification numbers) of KC1 and KC2 (using WA and WB matrices) are as follows:
[WPCMP∑] of WA (for KC1) = 1.1243e+003
[WPCMP∑] of WB (for KC2) = 1.2763e+003
[WPCMPmax] of WA (for KC1) =393.6130
[WPCMPmax] of WB (for KC2) =397.7265
Our method reports that chain 1 and 2 are non-isomorphic as the set of structural invariants [WPCMP∑] and [WPCMPmax] are different for both the KCs.

Matrix Representation of the Mechanisms of KC1

Fixing link 1, the first mechanism is developed. Hence, changing the diagonal element of first row and first column from 0 to 1 of [WPCM] matrix, the [WA-1] matrix is obtained and shown as

$$[WA\ \text{-}1]=\begin{pmatrix}
1 & 1.1 & 0 & 1.595 & 0 & 1.595 & 0 & 1.595 & 0 & 1.595 \\
1.1 & 0 & 1.595 & 0 & 1.595 & 0 & 1.595 & 0 & 1.595 & 0 \\
0 & 1.595 & 0 & 1.1 & 0 & 0 & 0 & 0 & 0 & 0 \\
1.595 & 0 & 1.1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1.595 & 0 & 0 & 0 & 1.1 & 0 & 0 & 0 & 0 \\
1.595 & 0 & 0 & 0 & 1.1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1.595 & 0 & 0 & 0 & 0 & 0 & 1.1 & 0 & 0 \\
1.595 & 0 & 0 & 0 & 0 & 0 & 1.1 & 0 & 0 & 0 \\
0 & 1.595 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.1 \\
1.595 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1. & 0
\end{pmatrix}$$

Structural Invariants of the First Mechanism of KC1

The Structural Invariants of first mechanism are derived from from the kinematic chains shown in Fig.1 obtained from the [WA-1] matrix are given as:
[WPCMP∑-1] = 1.2580e+003 and
[WPCMPmax-1] =393.6130
Similarly, changing the diagonal elements of rows and columns from 0 to 1 of [WPCM] matrix turn in turn, structural invariants of the other mechanism are obtained and given as:
[WPCMP∑-2] = 1.2580e+003 and
[WPCMPmax-2] =393.6130
[WPCMP∑-3] = 1.5393e+003 and
[WPCMPmax-3] =393.6130
[WPCMP∑-4] = 1.5393e+003 and
[WPCMPmax-4] =393.6130
[WPCMP∑-5] = 1.5393e+003 and
[WPCMPmax-5] =393.6130
[WPCMP∑-6] = 1.5393e+003 and
[WPCMPmax-6] =393.6130
[WPCMP∑-7] = 1.5393e+003 and
[WPCMPmax-7] =393.6130
[WPCMP∑-8] = 1.5393e+003 and
[WPCMPmax-8] =393.6130
[WPCMP∑-9] = 1.5393e+003 and
[WPCMPmax-9] =393.6130
[WPCMP∑-10] = 1.5393e+003 and
[WPCMPmax-10] =393.6130

Identification of the Distinct Mechanisms obtained from KC1 (Fig.1)

Observing the structural invariants for the above ten mechanisms, it is found that the structural invariants of link-1 and 2 are the same; hence they are structurally equivalent links and form only one distinct mechanism.

Similarly invariants of link-3, 4, 5, 6,7,8,9 and 10 are same; hence form second distinct mechanism.

Therefore, 2 DMs are obtained from KC1 shown in Fig.1

Structural Invariants of Mechanism of KC2

Using the above mentioned procedure, Structural Invariants of mechanisms obtained from KC2 may be found and are given as:

[WPCMP∑-1] = 1.3792e+003 and

[WPCMPmax-1] =397.7265
[WPCMP∑-2] = 1.3792e+003 and
[WPCMPmax-2] =397.7265
[WPCMP∑-3] = 1.5034e+003 and
[WPCMPmax-3] =366.3287
[WPCMP∑-4] = 1.5034e+003 and
[WPCMPmax-4] =366.3287
[WPCMP∑-5] = 1.5034e+003 and
[WPCMPmax-5] =366.3287
[WPCMP∑-6] = 1.5034e+003 and
[WPCMPmax-6] =366.3287
[WPCMP∑-7] = 1.5034e+003 and
[WPCMPmax-7] =366.3287
[WPCMP∑-8] = 1.5034e+003 and
[WPCMPmax-8] =366.3287
[WPCMP∑-9] = 1.8071e+003 and
[WPCMPmax-9] =488.1475
[WPCMP∑-10] = 1.8071e+003 and
[WPCMPmax-10] =488.1475

Here, link 1 and 2 are structurally equivalent link. Similarly link 3 to 8 are equivalent. Also link 9 and 10 are structurally equivalent link. Therefore 3 DMs can be derived from KC-2, as there are 3 different values of [WPCMP∑] i.e. 1.3792e + 003, 1.5034e + 003 and 1.8071e + 003 for the mechanisms obtained by fixing the link 1 to 10 in turn. Note that the result obtained by using other method [4], the same conclusion is obtained.

6.1 Example –2

The second example also concerns the mechanism KC of 10-bars as shown in Fig. 3.

$$WC1=\begin{pmatrix}
1 & 1.1458 & 0 & 0 & 0 & 0 & 1.5 & 1.5 & 0 & 1.375 \\
1.1458 & 0 & 1.1917 & 0 & 0 & 0 & 0 & 0 & 1.1917 & 0 \\
0 & 1.1917 & 0 & 1.1917 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1.1917 & 0 & 1.1917 & 0 & 0 & 1.1917 & 0 & 0 \\
0 & 0 & 0 & 1.1917 & 0 & 1.1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1.1 & 0 & 1.1 & 0 & 0 & 0 \\
1.5 & 0 & 0 & 0 & 0 & 1.1 & 0 & 0 & 0 & 0 \\
1.5 & 0 & 0 & 1.1917 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1.1917 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.1917 \\
1.375 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.1917 & 0
\end{pmatrix}$$

Structural Invariants of Mechanism

The structural invariants of the mechanisms are given as:
[WPCMP∑-1] = 640.2236
[WPCMP∑-2] = 805.8118
[WPCMP∑-3] = 822.4963
[WPCMP∑-4] = 752.8021
[WPCMP∑-5] = 788.4048
[WPCMP∑-6] = 790.6638
[WPCMP∑-7] = 768.2283
[WPCMP∑-8] = 786.8518
[WPCMP∑-9] = 819.4206
[WPCMP∑-10] = 837.1202
and
[WPCMPmax-1= 214.4093
[WPCMPmax-2= 196.3302
[WPCMPmax-3= 194.3133
[WPCMPmax-4= 202.0822
[WPCMPmax-5= 187.8990
[WPCMPmax-6= 190.4405
[WPCMPmax-7= 181.8486
[WPCMPmax-8= 186.8429
[WPCMPmax-9= 192.5296
[WPCMPmax-10= 183.0423

Our methods reports that 10 DMs can be derived from this chain as there are 10 different values of [WPCMP∑] of the inversion mechanism of KC- 3.

6.2 Example –3 (Identification of Co-spectral graphs)

The non-isomorphic kinematic chains have the same characteristic polynomials using (0, 1) adjacency matrices and their kinematic graphs are called as Co-spectral graphs. But the characteristic polynomials of such chains derived from [WPCM] matrices are distinct. Therefore, the structural invariants **[WPCMP∑] and [WPCMPmax]** are also distinct. This is proved with the help of examples of two kinemtic chains (KC4 and KC5) with 10 bars, 12 joints, three degree of freedom as shown in Fig 4 and Fig.5. The task is to examine whether these two chains are isomorphic. [W4] and [W5] represent the [**WPCM**] matrices while [W41] and [W51] represent the Link Adjancy matrices for these chains respectively.
The structural invariants using [WPCM] matrices [W4] and [W5] are as follows:
For (KC4):
[WPCMP∑] = 777.3699,
[**WPCMPmax**] = 350.4603
For (KC5):
[WPCMP∑] = 886.6169,
[**WPCMPmax**] = 389.4842

Our method reports that chains KC4 and KC5 are non-isomorphic as the set of values of **[WPCMP∑] and [WPCMPmax]** are different for both the kinematic chains. Note that by using method [8], the same conclusion is obtained.

But the same Structural Invariants values using Adjacency matrices [W41] and [W51] are as follows:
For (KC4):
[WPCMP∑] = 159.0000, [**WPCMPmax**] =68.0000
For (KC5):
[WPCMP∑] = 159.0000, [**WPCMPmax**] = 68.0000

$$W4 = \begin{pmatrix} 0 & 1.43 & 0 & 0 & 0 & 0 & 0 & 1.1 & 0 & 1.43 \\ 1.43 & 0 & 1.1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1.1 & 0 & 1.1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1.1 & 0 & 1.43 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.43 & 0 & 1 & 0 & 0 & 0 & 1.43 \\ 0 & 0 & 0 & 0 & 1.1 & 0 & 1.43 & 0 & 1.43 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1.43 & 0 & 1.43 & 0 & 0 \\ 1.1 & 0 & 0 & 0 & 0 & 0 & 1.43 & 0 & 1.43 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1.43 & 0 & 1.43 & 0 & 0 \\ 1.43 & 0 & 0 & 0 & 1.43 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$W5 = \begin{pmatrix} 0 & 1.43 & 0 & 0 & 0 & 0 & 0 & 1.43 & 1.43 & 0 \\ 1.43 & 0 & 1.1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1.1 & 0 & 1.43 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1.43 & 0 & 1.43 & 0 & 0 & 0 & 0 & 1.43 \\ 0 & 0 & 0 & 1.43 & 0 & 1.43 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1.43 & 0 & 1.43 & 0 & 0 & 1.1 \\ 0 & 0 & 0 & 0 & 0 & 1.43 & 0 & 1.1 & 0 & 0 \\ 1.43 & 0 & 0 & 0 & 0 & 0 & 1.1 & 0 & 0 & 0 \\ 1.43 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.43 \\ 0 & 0 & 0 & 1.1 & 0 & 1.1 & 0 & 0 & 1.43 & 0 \end{pmatrix}$$

$$W41 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$W51 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \end{pmatrix}$$

It means chains KC4 and KC5 should be isomorphic using Link Adjacency matrices. But the same Structural Invariants values using [WPCM] matrix are unique to clearly identify the KC with co-spectral graphs. The proposed [WPCM] matrix provides distinct set of characteristic polynomial coefficients of the kinematic chain with co-spectral graph also.

7. Conclusion

In the proposed method, the [PCM] matrix is able to distinguish the type of KP between two links. The [WPCM] matrix is derived from [PCM] matrix, which has the additional information about the type of links that are directly connected in the form of mutual interactive effect of the relative weights. The two structural invariants [WPCMP∑] and [WPCMPmax] are derived from the [WPCM]. These invariants are able to detect isomorphism among the mechanism KC and even the KC with co-spectral graph. However, the authors do not claim that this method presents an exhaustive study of the identification of the DM of KC. More work is needed before a final “best” form is chosen. However, it is believed that this paper presents a new concept on which a new identification system for distinct mechanism can be based. Such a new identification system would be extremely selective and would minimize, if not completely eliminate the possibility of duplicate identification for structurally different mechanisms.

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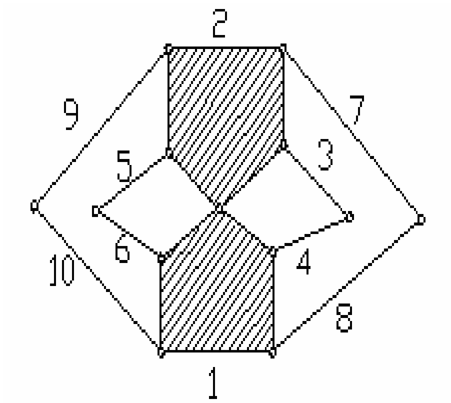

Fig.1

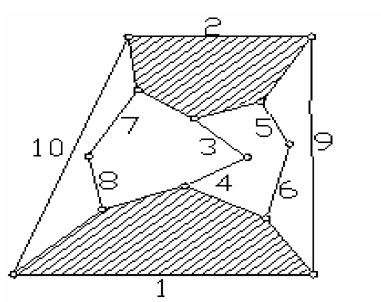

Fig.2

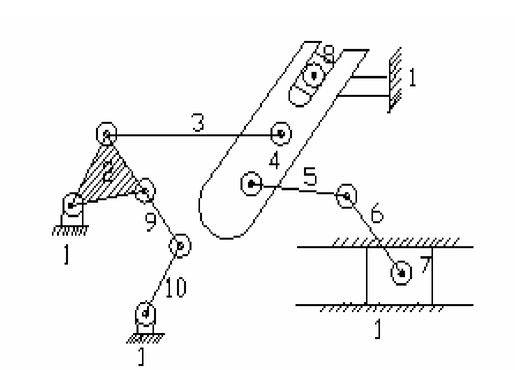

Fig.3

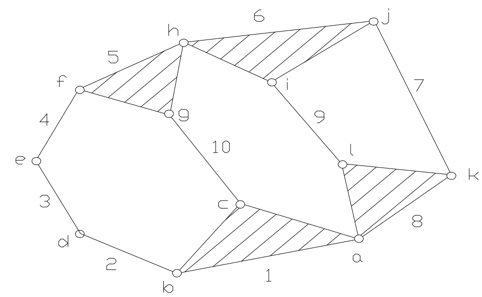

Fig.4: Ten bar chain,
three freedom

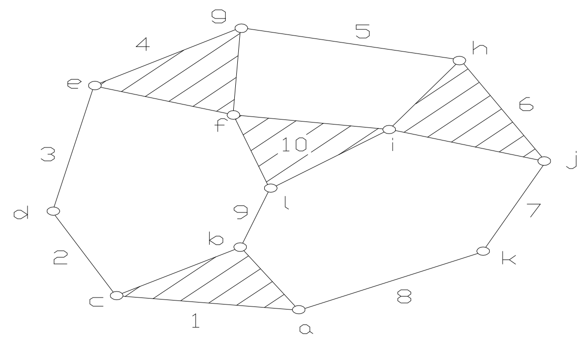

Fig.5: Ten bar chain,
three freedom