FSP Synthesis of an off-set five bar-slider mechanism with variable topology

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Abstract

A method to synthesize an off-set five-bar slider mechanism with variable topology is suggested. Synthesis is carried out in two phases for function generation for three finitely separated positions in phase-I and two finitely separated positions in phase-II. A dyadic complex number method is used to write the equations of motions. Numerical examples are provided.

Keywords: five-bar slider, variable topology, complex numbers

1 Introduction

Four bar-slider crank linkages have been widely used in industrial applications. Current work on five-bar linkages is mainly focused on revolute joint type. Little work has been done on five-bar slider crank linkages. Five bar-slider crank parallel manipulators can be used as effective singularity-free path generators if their dimensions and slider input ranges are properly selected. [1]. In this paper an off-set five bar-slider mechanism is synthesized by variable topology method using complex numbers.

A planar off-set five bar-slider mechanism has two degrees of freedom; it has one rotary and one linear independent as input shown in fig.1. A planar off-set five bar-slider mechanism with variable topology is a mechanism of two degrees of freedom that operates in two phases. During Phase-I of operation a link adjacent to the permanently fixed link i.e. crank O_aA [fig.2] is temporarily fixed and that result in an off-set single slider crank mechanism of one degree of freedom. The Synthesis procedure is then carried out for three finitely separated positions for the function generation task. The unknown parameters determined in Phase-I synthesis are considered while solving the remaining dimensions in Phase-II [2].

In Phase–II the slider is temporarily fixed in position 3 and the crank is released [fig3]. The resulting mechanism is a four bar mechanism of single degree of freedom. Now the synthesis is carried out for the determination of the other unknown parameters [2, 3 and 4].

To start with an overview of the variable topology mechanism is given to form the basis of the method developed in the present work. Rose [6] made indirect reference of five-bar variable topology mechanism with the help of graphical methods. Rawat [7] established a synthesis technique for five-bar topology mechanism operating in two phases. Joshi et.al. [8] used graphical synthesis of a five-bar slider variable topology mechanism. Balli and Chand [3] [4] [5] deal with aspects like transmission angle control, defects and solutions rectification of five-bar variable topology mechanism. Chand and Balli [2] proposed a method of synthesis of a seven-link mechanism with variable topology. Gadad, Daivagna and Balli presented combined triad and dyad synthesis of seven-link variable topology mechanism [9].Daivagna and Balli presented synthesis of five-bar slider with variable topology [10].

W. Z Guo et. al [11] deal with a new type of controllable mechanical press for metal forming applications. Abhijit Nagchaudhuri [12] deals with dynamic modeling and analysis of a crank slider mechanism. H.Zhou et.al [1] deal the path generation with singularity avoidance for five-bar slider-crank parallel manipulators. Eres Soylemez [13] dealt with Classical transmission angle problem for slider- crank mechanisms.

In this paper an off-set five-bar slider with variable topology is synthesized. A variable topology synthesis method is suggested as a tool to the graphical method of synthesis suggested by Joshi and Amaranth [8]. Many methods of synthesis like algebraic method, loop closure technique and graphical methods are available for the off-set single slider crank mechanisms and four bar mechanisms. The method of variable topology suggested in this paper reduces the cumbersome calculations of the algebraic methods. It has an increased accuracy over the graphical methods.

2 Variable Topology of Five-Bar Slider Mechanism

A planar off-set five-bar slider mechanism has two degrees of freedom. It is to be synthesized to perform three-position function generation in Phase-I and two-position function generation in Phase-II.

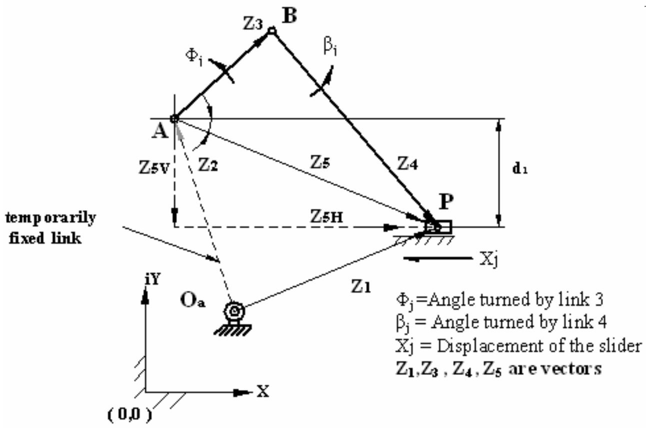

Figure 1: Off-set Five-bar slider Mechanism

2.1 Phase-I

Three positions of the slider is considered for the synthesis of the unknown parameters $\mathbf{Z}_3$, $\mathbf{Z}_4$ and $\mathbf{Z}_5$. Link O_aA is fixed temporarily and the line of action of the slider is off-set at a distance of d_1 from the temporarily fixed pivot A. Here off-set five-bar slider VTM of two-degree freedom becomes off-set slider crank mechanism of one-degree freedom. Consider the trace point as P, when the slider moves from P_1 to P_2 and from P_1 to P_3, the link AB moves through the angles ϕ_{12} and ϕ_{13} respectively. When the slider moves to position P_3, further motion is ceased. The slider does not move further. In this position the initial position of the Phase-II is considered.

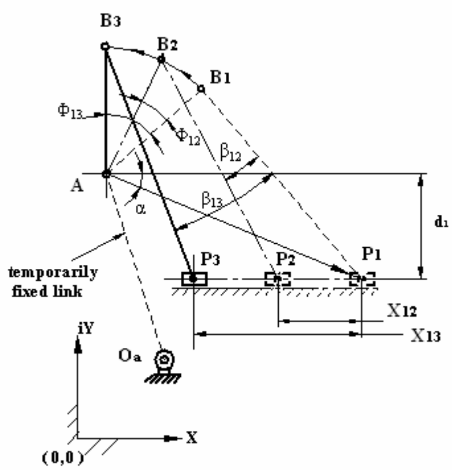

Figure 2: Off-set Five-bar slider mechanism with variable topology in phase-I

2.2 Phase-II

In Phase-II, the slider is fixed temporarily at P_3 at an off-set distance d_2 from the permanently fixed pivot O_a and the crank O_aA is released, resulting mechanism is a four bar mechanism of single degree of freedom. The input link i.e. the crank O_aA moves through an angle θ_{34}, link PB moves through an angle β_{34}. The trace point B moves from B_3 to B_4.

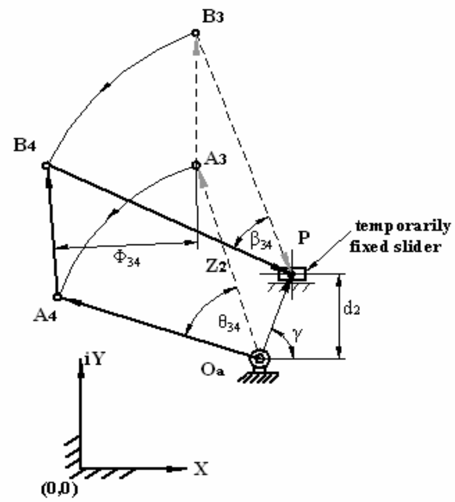

Figure 3: Off-set Five-bar slider mechanism with variable topology in phase-II

3 Synthesis

It is required to synthesize a planar five-bar slider variable topology mechanism; one can have two options as follows:

(i) One end i.e. the crank is fixed temporarily.

(ii) The slider is fixed temporarily.

The conventions to be followed to denote angles in Phase-I and in Phase-II are described in Table No.1. and Table No.2

Table 1: Parameters used in Phase I

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Links in terms of Vectors	*Phase-I from 1^{st} Position to 2^{rd} Position & from Position 1^{st} to 3^{rd} Position*
$O_aA = \mathbf{Z}_2$	Temporarily fixed link
$AB = \mathbf{Z}_3$	Angles Φ_{12}, Φ_{13}. (Given parameters)
$BP = \mathbf{Z}_4$	Angles β_{12}, β_{13} (Assumed parameters)
$AP = \mathbf{Z}_5$	Vector from A to P
$O_aP = \mathbf{Z}_1$	Vector from fixed point O_a to Slider position.
$\rho_{12} = \mathbf{Z}_{5H} + X_{12} / \mathbf{Z}_{5H}$ $\rho_{13} = \mathbf{Z}_{5H} + X_{13} / \mathbf{Z}_{5H}$	ρ_{12} & ρ_{13} are Stretch ratios X_{12} & X_{13} (prescribed parameters)
Number of solutions	∞^2

Table 2: Parameters used in Phase II

Links in Vectors	*Phase-II from 3rd Position to 4th Position*
$O_aA = \mathbf{Z}_2$	θ_{34} (prescribed parameter)
AB= $\mathbf{Z_3}$	ϕ_{34} (Assumed parameters)
BP = $\mathbf{Z}_4$	β_{34} (prescribed parameter)
AP = $\mathbf{Z}_6$	Vector from fixed point O_a to Slider position P_3
Number of solutions	∞^1
Total Number of solutions	∞^3

Phase-I and Phase-II described in Table-1 and Table 2 are used to write the dyad equations of the displacements of the slider for function generation.

A function generation mechanism is a linkage in which relative motion or a force between links connected to ground is taken into account. It is required to coordinate the sliding motion of the input i.e. the slider for three specified design positions to the angular position of the out put link AB [15, 16].

4.1 Phase-I Synthesis

In function generation problem, the input and output motions (X_{12} , ϕ_{12} , X_{13} , ϕ_{13}) are prescribed. β_{12} and β_{13} are free choices. Therefore there will be ∞^2 number of solutions. Then the unknowns $\mathbf{Z_3}$ and $\mathbf{Z}_4$ and $\mathbf{Z_5}$ are determined in Phase-I as follows:

Using loop closure vector and dyad equations and P as the tracing point we write:

$$\mathbf{Z_5} - \mathbf{Z_{5V}} - \mathbf{Z_{5H}} = 0 \tag{1}$$

$$\mathbf{Z_3} + \mathbf{Z_4} - \mathbf{Z_{5V}} - \mathbf{Z_{5H}} = 0 \tag{2}$$

$$\mathbf{Z_3}\left(e^{i\phi_j}\right) + \mathbf{Z_4}\left(e^{i\beta_j}\right) - \mathbf{Z_{5V}} - \mathbf{Z_{5H}}\rho_j = 0 \tag{3}$$

Subtracting (3) from (2) ,we get

$$\mathbf{Z_3}\left(e^{i\phi_j} - 1\right) + \mathbf{Z_4}\left(e^{i\beta_j} - 1\right) - \mathbf{Z_{5V}} - \mathbf{Z_{5H}}\rho_j = 0 \tag{4}$$

Writing equation (4) for 3 positions of the slider, we have,

$$\mathbf{Z_3}\left(e^{i\phi_{12}} - 1\right) + \mathbf{Z_4}\left(e^{i\beta_{12}} - 1\right) - \mathbf{Z_{5H}}\rho_{12} = 0 \tag{5}$$

$$\mathbf{Z_3}\left(e^{i\phi_{13}} - 1\right) + \mathbf{Z_4}\left(e^{i\beta_{13}} - 1\right) - \mathbf{Z_{5H}}\rho_{13} = 0 \tag{6}$$

Re writing in the following form,

$$\mathbf{Z_3}\left(e^{i\phi_{12}} - 1\right) + \mathbf{Z_4}\left(e^{i\beta_{12}} - 1\right) = \mathbf{Z_{5H}}\rho_{12} \tag{7}$$

$$\mathbf{Z_3}\left(e^{i\phi_{13}} - 1\right) + \mathbf{Z_4}\left(e^{i\beta_{13}} - 1\right) = \mathbf{Z_{5H}}\rho_{13} \tag{8}$$

Writing in the matrix form,

$$\begin{bmatrix} \left(e^{i\phi_{12}} - 1\right) & \left(e^{1\beta_{12}} - 1\right) \\ \left(e^{i\phi_{13}} - 1\right) & \left(e^{i\beta_{13}} - 1\right) \end{bmatrix} \begin{bmatrix} \mathbf{Z_3} \\ \mathbf{Z_4} \end{bmatrix} = \begin{bmatrix} X_{12} \\ X_{13} \end{bmatrix} \tag{9}$$

$$Let \delta = \begin{bmatrix} \left(e^{i\phi_{12}} - 1\right) & \left(e^{1\beta_{12}} - 1\right) \\ \left(e^{i\phi_{13}} - 1\right) & \left(e^{i\beta_{13}} - 1\right) \end{bmatrix} \tag{10}$$

Then, $\mathbf{Z_3}$ and $\mathbf{Z}_4$ are determined as followed

$$\mathbf{Z_3} = \begin{bmatrix} X_{12} & \left(e^{1\beta_{12}} - 1\right) \\ X_{13} & \left(e^{i\beta_{13}} - 1\right) \end{bmatrix} / \ \delta \tag{11}$$

$$\mathbf{Z}_4 = \begin{bmatrix} \left(e^{i\phi_{12}} - 1\right) & X_{12} \\ \left(e^{i\phi_{13}} - 1\right) & X_{13} \end{bmatrix} / \ \delta \tag{12}$$

From loop closure equation we have,

$$\mathbf{Z_3} + \mathbf{Z_4} = \mathbf{Z_5} \tag{13}$$

$\mathbf{Z_5}$ is determined .

The constant vector $\mathbf{Z_{5V}}$ for which magnitude is equal to the off-set distance d_1 and the variable vector $\mathbf{Z_{5H}}$ are determined as follows.

$$\mathbf{Z_{5H}} = -\mathbf{Z_5}\ e^{i\alpha}\ \mathrm{Cos}\,\alpha \tag{14}$$

$$\mathbf{Z_{5V}} = \mathbf{Z_5}\ e^{i\alpha}\ \mathrm{Sin}\,\alpha \tag{15}$$

d_1 = magnitude of $\mathbf{Z_{5V}}$

Where α = angle made by the vector $\mathbf{Z_5}$ with X-axis as shown in fig 2 Thus in phase-I, following vectors are determined.

$\mathbf{Z_3}$, $\mathbf{Z}_4$, $\mathbf{Z_{5V}}$ and $\mathbf{Z_{5H}}$

4.2 Phase-II Synthesis

When the mechanism moves from position 1 to position 3, it stops and switches to Phase-II. Here OaA is released and the slider is temporarily fixed. Now the mechanism becomes a four bar mechanism of single degree of freedom. Here the input angle is θ_{34}, the out put link PB describes β_{34}. It is required to coordinate the angular motion between the input link 2 and the output link 4 for the function generation. Hence the prescribed parameters are β_{34} and θ_{34}. Assuming the coupler angle ϕ_{34} for the link 3, the only unknown $\mathbf{Z}_2$ i.e. the link O_aA and $\mathbf{Z}_1$ are determined for two position function generation by taking the tracer point as B. The number of solutions is ∞^1 . Hence total number of solutions will be solutions. ∞^3

Writing dyadic equations, we have

$$\mathbf{Z}_2\left(e^{i\theta_{34}} - 1\right) + \mathbf{Z_3}\left(e^{1\phi_{34}} - 1\right) = \mathbf{Z}_4\left(e^{i\beta_{34}-1}\right) \tag{16}$$

Here the only unknown is $\mathbf{Z}_2$ and it is determined.

$$\mathbf{Z}_2 = \frac{\mathbf{Z}_4\left(e^{i\beta_{34}-1}\right) - \mathbf{Z_3}\left(e^{1\phi_{34}} - 1\right)}{\left(e^{i\theta_{34}} - 1\right)} \tag{17}$$

By loop closure equation, we have,

$$\mathbf{Z}_2 + \mathbf{Z_3}\left(e^{1\phi_{13}}\right) + \mathbf{Z}_4\left(e^{i\beta_{13}}\right) - \mathbf{Z}_6 = 0 \tag{18}$$

Hence $\mathbf{Z}_6 = \mathbf{Z}_2 + \mathbf{Z_3}\left(e^{1\phi_{13}}\right) + \mathbf{Z}_4\left(e^{i\beta_{13}}\right)$ (19)

The off-set distance d_2 is determined as following.

$$d_2 = \mathbf{Z}_6 \otimes \left(e^{i(90-\gamma)}\right) \tag{20}$$

Where, γ is the angle made by the vector $\mathbf{Z}_6$ with x-axis.

Also, $\mathbf{Z}_1$ Vector is determined by the closed loop vector equation as given in (21).

$$\mathbf{Z}_1 = \mathbf{Z}_2 + \mathbf{Z_3} + \mathbf{Z}_4 \tag{21}$$

Hence all the design parameters of the off-set five-bar slider mechanism are determined from Phase-I and Phase-II.

5. Advantages and Limitations

5.1 Advantages

Simplicity and generality are the attractions of the method as compared to other method of synthesis of five bar-slider mechanisms. Unlike graphical method, it is not limited by drawing accuracy. Since the dimensions determined in phase-I are carried to phase-II leading to less calculations. The method can be applied to synthesis of mechanisms for three; four or five infinitesimally separated positions.

5.2 Limitations

The proposed method is applicable only to complex number approach. The synthesized mechanism may suffer from branch, order, Grashof or circuit defects which can be rectified separately.

6. Conclusions and Discussions

The present work suggests variable topology method using dyad techniques for synthesizing a five-bar slider. An analytical method for synthesizing a five-bar slider crank with variable topology for three positions in Phase-I and two positions in Phase-II is suggested for function generation. Complex numbers, which readily lend themselves as an ideal tool for modeling linkage members as parts of planar mechanisms, are used for writing displacement equations for dyads. The coupler point B is displaced from position one to position four that may be used for the application of transfer of object from place one to place four. The mechanism may suffer from branch, order, Grashof or circuit defects which can be rectified separately. As this is synthesized work, the authors will consider the study of the performance of the mechanism in their future work.

Examples

6.1 Example 1:

It is required to synthesize a Five Bar slider with variable topology shown in Fig.1. Given that $\phi_{12} = 22^0$ CCW, ϕ_{13} =45.5^0 CCW, X_{12} =25.0mm, X_{13} =50.0mm for phase-I and θ_{34} =40^0 CCW, β_{34} =29^0 CCW for the phase-II.

6.1.1 Solution:
Phase-I
Assuming $\beta_{12} = -12.76^0$, $\beta_{13} = -21.3^0$,
From equation (11) we get,
$\mathbf{Z_3} = (28.9613 + 24.4669i) = 37.9129$ mm $\angle\ 40.1916^0$
From equation (12),
$\mathbf{Z}_4 = 47.4099–56.8432i = 74.0192$ mm $\angle\ 309.8300^0$

Similarly from equation (13) ,
$\mathbf{Z_5} = -76.3713 - 32.3762i = 82.9505$ mm $\angle\ 337.0264^0$.
The offset distance is calculated as 31.00mm using equation (14).
Phase-II Assuming $\phi_{34} = -7^0$, from equation (17) we get ,
$\mathbf{Z}_2 = -17.2488+55.5519i = 58.1682$ mm $\angle 107.1682^0$
Again from the equation of loop closure (21), we get
$\mathbf{Z}_1 = -59.1225 + 23.1758i = 63.4026$ mm $\angle 21.40^0$
The synthesized mechanism is shown in figure 4.

6.2 Example 2:

It is required to synthesize a Five Bar slider with variable topology shown in Fig.1. Given that ϕ_{12} = 17.87^0 CCW, ϕ_{13} =35.33^0 CCW, X_{12} =30.0mm X_{13} =60.0mm for phase-I and θ_{34} =63.12^0 CCW, β_{34} =48.16^0 CCW for the phase-II.

6.2.1 Solution:
Phase-I
Assuming $\beta_{12} = -8.73^0$, $\beta_{13} = -15.67^0$,
From equation (11) we get,
$\mathbf{Z_3} = (11.4606 + 15.0420i) = 18.9104$ mm $\angle\ 52.6960^0$
From equation (12)
$\mathbf{Z}_4 = 29.4697–54.1962i = 61.6903$ mm $\angle\ 308.5356^0$
Similarly from equation (13)
$\mathbf{Z_5} = -40.9302 - 39.1542i = 56.6421$ mm $\angle\ 316.27^0$.
The offset distance is calculated as 33.24mm using equation (14).

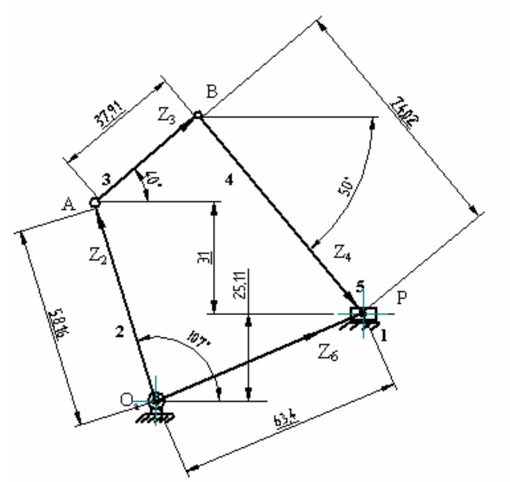

Figure 4: Synthesized off-set Five Slider Mechanism with variable topology for the example 1

Phase-II Assuming $\phi_{34} = 2.14^0$, from equation (17) we get $\mathbf{Z}_2 = --4.7491+47.2765i = 47.5145 \text{ mm } \angle\ 95.74^0$

Again from the equation of loop closure (21), we get
$\mathbf{Z}_1 = 7.0465\text{mm } \angle\ 58.9136^0$

The synthesized mechanism is shown in figure 5. The function curves for phase-I and phase-II for the examples are shown in figure 6 and figure 7 respectively.

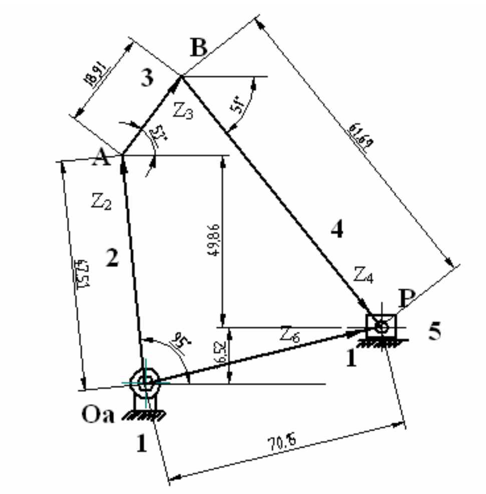

Figure 5: Synthesized off-set Five Slider Mechanism with variable topology for the example 2

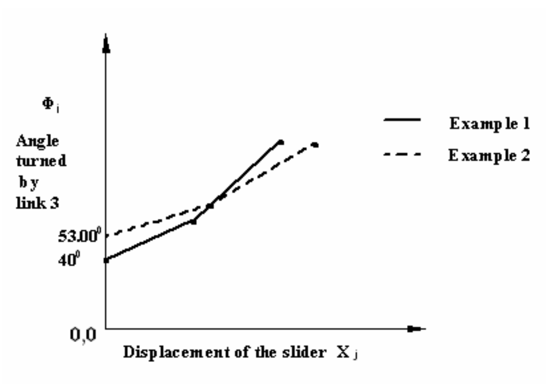

Figure 6: Function curves for phase-I

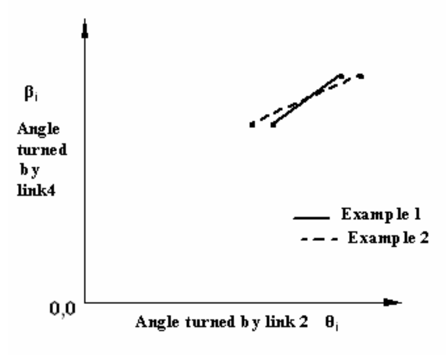

Figure 7: Function curves for phase-II

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