Distributed Parameter Model Based Trajectory Tracking Control of a Single-Link Flexible Robot Manipulator

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Abstract

 In this paper we present a new approach for joint trajectory tracking of a single-link flexible robot manipulator. Firstly a distributed parameter dynamic model is derived using extended Hamilton Principle. Based on this model a new moment feedback trajectory tracking control scheme is designed for a single-link flexible robot manipulator having a payload at its free end. The control scheme is proved to be stable using Lyapunov approach and the tracking error and vibrations of link are bounded. The effectiveness of the control scheme is demonstrated using simulation studies.

Keywords: Flexible-Link Manipulator, Distributed Parameter Model, Hamilton principle, Lyapunov Stability.

1 Introduction

Modeling and vibration control of robots with flexible links have found great motivation due to many advantages over the rigid-link ones such as lower arm cost, better energy efficiencies, higher operation speed and improved mobility. Moreover use of light weight robots where link flexibility is an inherent property, is preferred in many applications as use of bulky rigid robots is not suitable for those applications e.g. space applications.

In recent years, extensive research has been carried out for the control of flexible-link robots. As the dynamic behavior of a flexible-link is described by a partial differential equation, the system is a distributed parameter system and possesses infinite number of dimensions, which makes it difficult to control [1]. As a consequence, the control of flexible robots has been considered as one of the most challenging problems in the control community.

In order to derive tractable solutions to the original problem approximate finite dimensional models are widely used in research community. Two most commonly used approaches namely assumed modes method and finite elements method have been very popular in the control community. Cannon and Schmitz [2] used assumed modes method in order to solve problems related to single flexible link manipulators, considering that the sensors are not collocated with actuators. Book [3] used Lagrangian formulation to model flexible link manipulators where the link deflection is assumed to be small so that the link transformation is represented in terms of additional modal shapes. Other applications of modal approximations are found in [4, 5]. Serna and Bayo [6] presented an efficient method for describing the dynamics of flexible robots based on Lagrangian formulation and finite element method. Pereira and Proenca [7] analyzed the flexible structure using a finite element model strongly reduced by applying the component mode synthesis technique. Usoro et.al. [8] also used finite element approach to approximate the original system.

However these approaches have attracted much attention, many problems arise due to approximation factor e.g. excitation of high frequency dynamics, requirement of relatively higher order controller to achieve high accuracy of performance etc. An alternative approach to derive controllers directly from PDE or controllers based on distributed parameter dynamic models, has also been popular in the design of control schemes. Morguel [9] proposed a regulating control of Euler–Bernoulli beam using a boundary force and moment at the free end in addition to joint torque. Luo [10] proposed an asymptotically stable strain feedback control for a regulating control of link vibration. Ge et al. [11] designed a strain feedback regulating control, where the payload mass was considered. De Queiroz et al. [12] proposed an adaptive set point control scheme using joint torque and a boundary force at the free end.

For the approximation approaches, various fully developed control theories are available. However, due to the complexities of the distributed-parameter dynamic model, most of the research efforts have addressed the regulation of control systems around a goal position. The resulting regulating control schemes may not be used independently in trajectory control. In this paper we present a new control design scheme for joint trajectory tracking of a single link flexible robot having a payload at its free end. The proposed control is stable for trajectory tracking which is proved using Lyapunov approach. Shifman [17] derived a controller for Euler Bernoulli beam using the complete distributed model along with a Lyapunon function approach while the model structure was preserved in the control law. Ge et al [18] presented an asymptotically stable end point regulation of a flexible scara/Cartesian robot and they proved the closed loop stability for the original distributed parameter system using Lyapunov function approach

The paper is organized as follows. In section 2 the dynamic model of the system is derived using extended Hamilton principle. The proposed controller is presented in section 3. Section 4 deals simulation studies. Concluding remarks are given in section 5.

2 Dynamic Modeling

We consider a one-link flexible robot manipulator which is rotated by a motor in a horizontal plane. The flexible-link of length L, having uniform mass density ρ per unit length, uniform flexible rigidity EI, is clamped on the vertical shaft of the motor at the one end and has a concentrated mass M_p, at the other end. Let I_h, be the moment of inertia of the rotor of the motor. Let $\theta(t)$, be the angle of rotation of the motor. Let $\tau(t)$, be the torque developed by the motor. In Fig.(1) a pair (i, j) of orthogonal unit vectors, which are fixed at the vertical shaft of the rotor of the motor is shown. Let $w(x,t)$, denote the transverse displacement at time t, and at a spatial point x, and $w(L,t)$, denote the transverse displacement at the end point. Due to the planar motion and the slender link we assume the Euler-Bernoulli model, for which the rotary-inertia and shear deformation effects can be ignored.
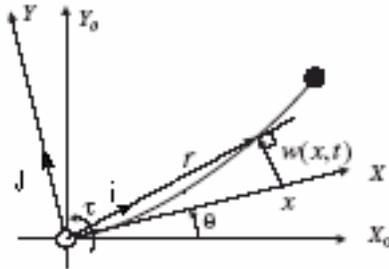
Figure 1: A flexible-link robot manipulator.

Let P be the position vector of the arm and r be the position vector of the flexible arm at a general position. The position vectors P, r and their time derivatives are given by

$$P = Li + w(L,t)j$$
$$\dot{P} = (L\dot{\theta}(t) + \dot{w}(L,t))j + w(L,t)\dot{\theta}(t)i$$
$$r = xi + w(x,t)j$$
$$\dot{r} = (r\dot{\theta}(t) + \dot{w}(x,t))j + w(x,t)\dot{\theta}(t)i$$

Where a dot denotes the time derivative.

Now, the total kinetic energy T and the potential energy V of the link is given as

$$T = \frac{1}{2}I_h\dot{\theta}^2 + \frac{1}{2}M_p\dot{P}^T\dot{P} + \frac{1}{2}\int_0^L \rho\,\dot{r}^T\dot{r}\,dx \qquad (1)$$

$$V = \frac{1}{2}\int_0^L EI\left[w''(x,t)\right]^2 dx \qquad (2)$$

The virtual work δW is given by

$$\delta W = \tau\delta\theta \qquad (3)$$

The law governing the motion of mechanical system is the extended Hamilton's principle which can be written as follows

$$\int_{t_2}^{t_1}(\delta T - \delta V + \delta W)dt = 0 \qquad (4)$$

Substituting Eq.s (1) to (3) in Eq. (4) and neglecting nonlinear terms $w^2(x,t)\dot{\theta}^2(t), w^2(L,t)\dot{\theta}^2(t)$ and then after simplification we get the following PDE's of the system dynamics as follows

$$(I_h + \frac{1}{3}\rho L^3)\ddot{\theta} + \rho\int_0^L x\,\ddot{w}(x,t)\,dx + M_pL(L\ddot{\theta} + \ddot{w}(L,t)) = \tau \qquad (5)$$

$$\rho[x\ddot{\theta} + \ddot{w}(x,t)] = -EI\,w''''(x,t) \qquad (6)$$

The base bending moment of the flexible link can be calculated by

$$EI\,w''(0,t) = -\frac{1}{3}\rho L^3\ddot{\theta} - \rho\int_0^L x\,\ddot{w}(x,t)\,dx -$$
$$- M_pL\left[L\ddot{\theta} + \ddot{w}(L,t)\right]$$

Thus the Eq. (5) can be reduced to

$$I_h\ddot{\theta} = \tau + EI\,w''(0,t) \qquad (7)$$

The corresponding boundary conditions are given by the following set of equations:

$$w(0,t) = 0 \qquad (8)$$
$$w'(0,t) = 0 \qquad (9)$$
$$w''(L,t) = 0 \qquad (10)$$
$$EI\,w'''(L,t) = M_p[L\ddot{\theta}(t) + \ddot{w}(L,t)] \qquad (11)$$

3 Controller Design

In this section, based on the distributed dynamic model and boundary conditions, a new trajectory control law will be designed. The Lyapunov stability theorem will be used as a mathematical tool.

Let the joint tracking error be defined as

$$r_\theta = \theta_d - \theta \qquad (12)$$

Where θ_d is a time varying desired trajectory of joint angle θ.

To design a trajectory tracking controller we shall make an assumption.

3.1 Assumption

The desired joint trajectory θ_d and its first and second time derivatives, $\dot{\theta}_d$ and $\ddot{\theta}_d$, are assumed to be uniformly bounded. In addition, $\dot{\theta}_d = 0$ and $\ddot{\theta}_d = 0$ are assumed for all time $t \geq t_m$ with some finite time $t_m \geq 0$.

Now consider a flexible-link robot whose dynamics is described by Eq.s (6) to (11). Suppose that the control

objective is to have the flexible-link robot track a desired trajectory satisfying the above stated assumption as accurately as possible while promptly suppressing the resulting link vibration asymptotically to zero. Then the following control law can achieve the objective while keeping all internal signals bounded.

$$\tau = I_h \ddot{\theta}_d + K_D \dot{r}_\theta + K_P r_\theta + K_1 \left| EIw_0'' \dot{\theta}_d \right| \dot{r}_\theta + K_2 EIw_0'' \quad (13)$$

Where K_D, K_P, K_1 and K_2 are positive control gains; $|.|$ is the absolute value of the corresponding variable.
Consider the Lyapunov function candidate $V_L(t)$:

$$V_L(t) = \frac{(1+K_2)}{2} \left[\int_0^L EIw''^2 \, dx + \int_0^L \rho(\dot{w} + x\dot{\theta})^2 \, dx + \right.$$
$$\left. + M_P (\dot{w}(L,t) + L\dot{\theta})^2 \right] + \frac{1}{2} (I_h \dot{r}_\theta^2 + K_P r_\theta^2) \quad (14)$$

The time derivative of $V_L(t)$ is given as

$$\dot{V}_L(t) = (1+K_2) \left[\int_0^L EI \, \dot{w}'' w'' dx + \int_0^L \rho(\ddot{w} + x\ddot{\theta})(\dot{w} + x\dot{\theta}) \, dx \right.$$
$$\left. + M_P (\ddot{w}(L,t) + L\ddot{\theta})(\dot{w}(L,t) + L\dot{\theta}) \right] + \dot{r}_\theta (I_h \ddot{r}_\theta + K_P r_\theta) \quad (15)$$

Now, $(\ddot{w} + x\ddot{\theta})$ is computed from Eq. (6) and is substituted into second integral. Then subsequent integrations by parts along with the boundary conditions Eq. (8) to (11) we get ,

$$\dot{V}_L(t) = -(1+K_2)EIw''(0,t)\dot{\theta} + \dot{r}_\theta (I_h \ddot{r}_\theta + K_P r_\theta) \quad (16)$$

Substituting the joint dynamics (7) with the control law (13) into $\ddot{r}_\theta$ yields

$$\dot{V}_L(t) = -(1+K_2) EIw''(0,t)\dot{\theta}_d - \left(K_D + K_1 \left| EIw''(0,t)\dot{\theta}_d \right| \right) \dot{\theta}_d^2$$

$$\dot{V}_L(t) \le (1+K_2) \left| EIw''(0,t)\dot{\theta}_d \right| - \left(K_D + K_1 \left| EIw''(0,t)\dot{\theta}_d \right| \right) \dot{\theta}_d^2$$

$$= -K_D \dot{r}_\theta^2 - K_1 \left| EIw''(0,t)\dot{\theta}_d \right| \left(\dot{r}_\theta^2 - \frac{(1+K_2)}{K_1} \right) \quad (17)$$

Then $\dot{V}_L(t) \le 0$ is guaranteed for all $\dot{r}_\theta$ satisfying $\dot{r}_\theta^2 \ge \frac{(1+K_2)}{K_1}$, which implies the boundedness of $V_L(t)$ and hence the boundedness of all internal signals $(\dot{r}_\theta, r_\theta, \dot{w}, w'' \in L_\infty)$. This also implies the boundedness of the control input τ defined in Eq. (13) since $\ddot{\theta}_d$ and $\dot{\theta}_d$ are uniformly bounded. Then the joint dynamics (7) gives $\ddot{\theta} \in L_\infty$.

Since $\dot{\theta}_d = 0$ for all $t \ge t_m$ with some finite time $t_m \ge 0$,

$$\dot{V}_L(t) \le -K_D \dot{r}_\theta^2$$

is guaranteed for all $t \ge t_m$. Then integration of the $\dot{V}_L(t)$ yields $\int_0^\infty \dot{r}_\theta^2 < \infty$ so that $\dot{r}_\theta \in L_2$. In addition, $\ddot{\theta} \in L_\infty$ is guaranteed as shown above. Then, as a consequence of Barbalat's lemma [14], $\dot{r}_\theta \to 0$ asymptotically as $t \to \infty$, which also implies $\ddot{r}_\theta \to 0$ asymptotically as $t \to \infty$, since $\ddot{r}_\theta \in L_\infty$.

Since $\dot{r}_\theta \to 0$ and $\ddot{r}_\theta \to 0$ as asymptotically as $t \to \infty$, , the joint dynamics (7) and the control law (13) result in $K_P r_\theta + (1+K_2)EIw''(0,t) \to 0$ asymptotically as. $t \to \infty$, Since $r_\theta \in L_\infty$ and $\dot{r}_\theta \to 0$ asymptotically as $t \to \infty$, θ and hence r_θ approach some finite constants, which implies $EI w''(0,t)$ also approach some finite constant. Since there is no applied force or moment on the link or payload other than the joint torque, $EI w''(0,t)$ (the moment of the link at $x = 0$) should approach zero in the steady state $(\dot{\theta} \to 0)$, which implies $r_\theta \to 0$ asymptotically as $t \to \infty$. Finally, $EI w''(0,t) = 0$ in the steady state implies zero deflection of the link. Therefore, it can be concluded that $w(x,t) \to 0$ asymptotically as $t \to \infty$.

4 Simulation Studies

In this section the numerical study of the proposed scheme is presented. The Table 1 describes the parametric values of flexible manipulator.

Table 1: Parameters of the Flexible Manipulator

Parameters	Values
Mass per unit length	0.45 kg
Link length	0.5 m
Payload mass	0.2 kg
Moment of inertia of rotor of motor	0.048 kg m^2
Flexural rigidity (EI)	1.99 Nm^2

The desired trajectory is chosen as

$$\theta_d = \frac{\pi}{2} \left[6\left(\frac{t}{t_m}\right)^5 - 15\left(\frac{t}{t_m}\right)^4 + 10\left(\frac{t}{t_m}\right)^3 \right]$$

Where $t_m = 10 \sec$. The initial values for joint angle are $\theta(0) = \frac{\pi}{10}, \dot{\theta}(t) = 0$. The gain coefficients in the proposed control law are taken as $K_d = 10, K_p = 2, K_1 = 1, K_2 = 1$. The whole system is simulated for 15 sec. The numerical solution of the differential equations of the system is carried out using neural network approach [15]. Using this approach the

joint angle and link deflection are assumed in the form of a feedforward neural network [16] as follows

$$\theta(t) = \frac{\pi}{10} + t^2 N_\theta(a,t)$$

$$w(x,t) = x^2(L-x)^3 N(b,t) + (3Lx^2 - x^3)N^*(c,t)$$

Where $N_\theta(a,t), N(b,t), N^*(c,t)$ represents feed forward neural networks and a, b, c represent their weights respectively. The whole interval is divided into 15 subintervals for solving the equations.

The following figures depict the performance of the designed controller and response of the system.
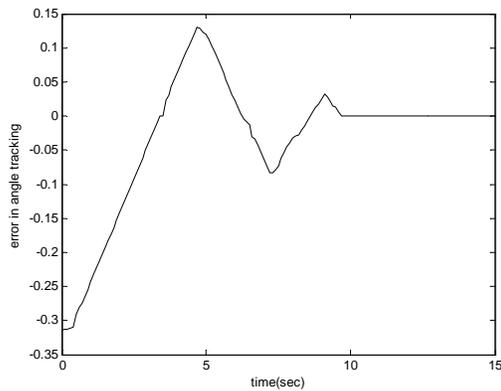
Figure 2: Error in trajectory tracking
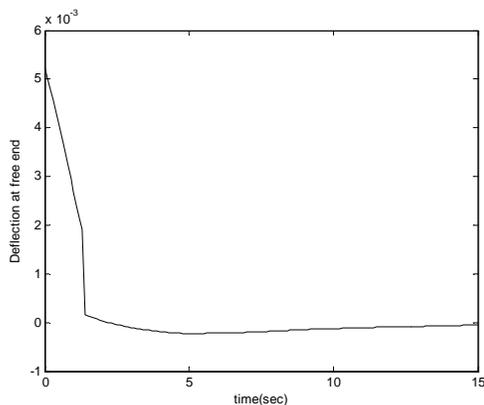
Figure 3: Deflection at free end

5 Conclusion

In this paper a new feedback joint trajectory control law has been presented based on distributed parameter dynamic model of a single flexible link robot manipulator. The proposed control scheme is stable throughout the entire trajectory tracking. In this scheme the drawbacks of truncated models are avoided and satisfactory results are achieved. The proposed control is considered in light of negligence of rotary inertia effects and axial forces. The present study will be extended to cases considering the above effects. As well as gravity effects and two link flexible robot manipulators will be considered for trajectory tracking problems.

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