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Relation of Part Positions and Kinematic Freedom: A Survey

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Abstract

Computer aided design of machines involves modeling and analysis of the articulated assemblies. An assembly model is created by relative positioning of parts, whereas a kinematic model is created by specifying kinematic constraints between the parts. Over 100 papers are reviewed in the areas of relative part positioning and deriving kinematic information from parts or assemblies. The papers are categorized by the approaches for the generation of assembly and kinematic models. The various representation schemes available for these two models are presented. It is observed that there is redundancy in the specification of assembly and kinematic constraints, and no comprehensive method is available to integrate these two models. Also, these two representations are inconsistent. The attempts made to bridge the gaps between these two models are discussed. The physical contact between the parts in an assembly reduces the degrees of freedom of the parts involved. This paper attempts to establish the relation between the relative part positions and the kinematic freedoms of the parts implicitly available in the literature.

Keywords: Part positioning, Degrees of freedom, Assembly constraints, Kinematic constraints.

1 Introduction

In physical assembly of rigid parts, the parts are positioned relative to each other. The positioning of parts causes some of the low level geometric entities like faces, edges, and vertices of the parts to be in contact. The entities in contact between the parts constrain the relative motion between them for a part can not penetrate through other parts in the assembly. The position of a part in space is uniquely defined by specifying its location and orientation with respect to some reference system. Three parameters are required to specify the location and another three parameters are required to specify the orientation. In kinematics, a rigid body in space has six degrees of freedom (DOF) representing the allowable motions of the part. Assembly models are created by fixing the positions of parts relative to each other, whereas kinematic models are created by specifying the allowed motions between the parts.

Three different relative positions P1, P2, and P3 of the parts Prt1 and Prt2 are shown in Figure 1(a). The entities that are in contact and the resulting allowed relative motions between the two parts are given in Figure 1(b). It can be seen that the allowed relative motion between the two parts changes with the relative positions and hence the entities in contact. Whereas, in any form closed kinematic joint, the relative position of the mating parts could change without affecting the kinematic freedoms between them.

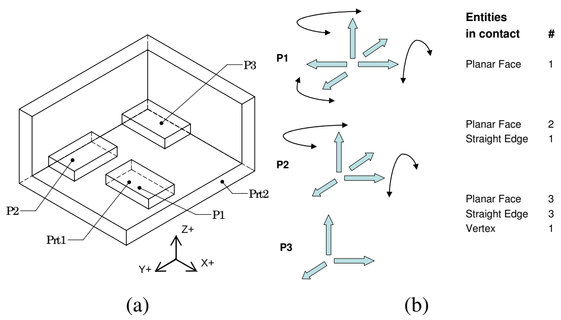

Figure 1: Relation between part positions and kinematic freedom.

The relative motion between parts depends upon the individual part geometries and their relative positioning. The earliest work on relating the geometry, position, and motion of the rigid bodies in contact is done by Reuleaux [1]. He identified the manifold of motions eliminated by each of the point contacts for the planar motion of the rigid body and thus led to determine the number of point contacts necessary to fix a rigid body. This approach was extended by Somov [2] to the general spatial case where the manifolds of screw motions (twists) eliminated by the various point contacts is considered. A method of relating the geometry of contacting surfaces to motion capability in the joints of spatial linkages is presented by Waldron [3] based on Ball's [4] screw system and reciprocal screw system. The relation between assembly and kinematic model is illustrated in Figure 2

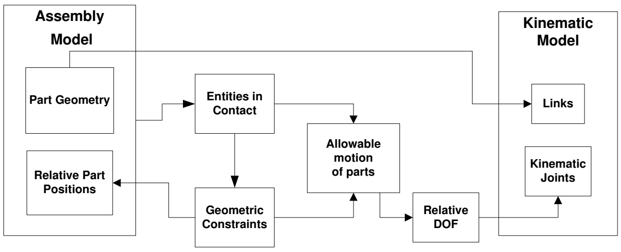

Figure 2: Relation between assembly and kinematic models.

2 Assembly Modeling: Part Positioning

A task common to both assembly modeling and kinematic analysis is the determination of part positions satisfying certain constraints between these parts. There are two categories of geometric assembly relationships: mating conditions and kinematic joints. The former is static, whereas the latter allows relative motion and holds despite changes in the components dimensions. An assembly constraint is created by mating conditions that control the ability of the components to assemble properly according to the design intent [5]. The various approaches for part positioning are discussed in the following.

2.1 Using Transformations

Part positioning in assembly involves specifying part location and orientation. It can be expressed relative to some global reference [6, 7] or with respect to other part/s [8, 9, 10, 11]. In many older assembly modeling systems, the parts are usually positioned by specifying a 4 x 4 homogeneous transformation matrix (T matrix) by the designer for each part. It is quite awkward and prone to error [12]. Eastman [8] developed a tree structure called *location graphs* whose vertices are shapes and edges are T-matrices relating the shapes. The shape is located relative to its ancestors and moves with them automatically. Wesley [13] also made use of T-matrices. A shape's T-matrix always transforms its coordinates into a global reference frame. Hence, the parts of a subassembly will not automatically move with it. The user must manually determine and input the T-matrices, and is also responsible for avoiding inconsistencies that arise from cycles in *location graphs*. Another approach to specify transformations uses Euler angles and a position vector from which a unique homogeneous T-matrix can be calculated. The three rotation angles and three scalar translations correspond to the six DOF of a rigid body [10].

2.2 Using Mating Relations

The assembly model can be obtained by specifying assembly constraints and solving them to find the positions of components. The methods for specifying assembly constraints are classified into geometry mating and joint mating approach, according to the type of objects on which the assembly constraints are specified [14].

In the joint mating approach [15, 16, 17, 14], the assembly constraints are specified on the component model itself. The joint mating constraints such as revolute and prismatic define the relations between components. Kim et al. [14] proposed a method that generates assembly models from kinematic joint constraints. The joint constraints are expressed in terms of the relations between components rather than relations between geometric elements. The joint coordinates or mating constraints need not to be specified in this approach. The commercial kinematic analysis packages like ADAMS [18] use joint mating approach, wherein the joints are defined using the markers attached to the parts. It requires the designer to manually specify joint coordinates as well as joint constraints. These tasks are error-prone and cumbersome for the designer. Moreover, this approach completely ignores the geometry of the parts.

In geometry mating approach, the geometric elements such as faces and axes are used to specify geometry mating constraints. Kim et al. [19] proposed a geometric constraint solving method that takes well-constrained geometric mating conditions such as "against" and "fits". This modeling approach is widely used in commercial modelers such as SolidWorks [20]. However, the mating tasks are very tedious and the designer is required to have a comprehensive understanding of mating constraints and geometric elements to specify mating conditions. Moreover, it is difficult to automatically update the assembly models after the component geometries are modified since the mating conditions are dependent on the geometries of the parts.

A common method for expressing physical constraints between parts is to specify the contact between pairs of planar, cylindrical, or spherical surfaces on adjacent parts, called mating conditions [10]. The set of mating relations between parts in an assembly is an implicit representation of the position of each part. The part-positioning problem involves resolving the mating relations to determine the positions of all parts, and thus the final configuration of the assembly. This problem is termed as a geometric constraint satisfaction. There are two approaches to solve this problem. In first approach, a set of nonlinear algebraic equations are formed and then solved using symbolic algebra [21, 22, 23, 24, 25, 26] or by using iterative numerical methods [27, 28, 7, 29, 30, 31, 16]. In second approach, a symbolic geometric reasoning is employed to find part positions. This approach is based on DOF analysis [15, 32, 33, 34].

Ambler and Popplestone [21, 22] set up algebraic equations for assembly relations - *fits* (axis-coincidence) and *against* (planar-contact), using the relative DOF as the variables for the equations. They described a symbolic algebra to solve a system of such equations, and thus determined the effect of the relations on the positions of bodies. The form of the solution identifies the DOF variables that remain unconstrained. While the method is general, the symbolic solution process is slow, and it may not always be successful. They proposed the use of faster lookup-table procedures [35] in place of the slower symbolic computation.

Lee and Andrews [7] computed the T-matrix in the world coordinate system for each part after all mating conditions had been interactively assigned. However, special procedures are needed to solve for the rotational component of each part's configuration. The 12 variable coefficients of the T-matrix form the unknowns in a system of equalities used to establish the positions of the parts. Equality constraints are generated for the against and fits relations. Since the relations usually impose more equality constraints than there are variables, the authors first eliminate linearly dependent equations, and then arbitrarily choose a set of equations that are equal in number to the number of variables. The reduced set of equations are solved simultaneously using the Newton-Raphson method. Results obtained from this approach are slow, and dependent on the choice of the correct equations.

Rossignac [36] proposed an approach to infer the positions of parts in an assembly based on rigid motions that are stored in the form of unevaluated constraints on the boundary elements of the part models. The problem of converting the constraints into a large system of equations is avoided in this approach. However, the user is responsible for designing a sequence of rigid motions to satisfy the constraints. Also, multiple mating feature relationships between pairs of parts are not allowed. A sequential solution of geometric constraints is suggested using four types of motions.

Rocheleau and Lee [29] created an assembly model by interactively specifying *against* and *fits* mating conditions between the individual components. Newton-Raphson iteration in conjunction with least-squares technique is used to solve the system of equations, saving on the initial overhead of selecting the correct subset of equations. The computational saving compared to the previous work is significant; however, a large number of simultaneous equations may be set up for strongly connected assemblies having a number of parts. Only two types of mating conditions *against* and *fits* are incorporated. It is suggested that kinematic analysis can be performed directly from an assembly model but no procedure is outlined. Oliver and Harangozo [16] combined the interactive modeling approach [29] with kinematic position analysis to develop a method of inferring link positions for planar mechanisms containing closed kinematic loops. The sequential and simultaneous position solution strategies are partially combined.

The numerical-solution strategies discussed above are sensitive to the initial starting values of the unknowns, and they may fail to converge, or yield incorrect locally optimum solutions. The constraint equations are set up for all the relations in the assembly, and are solved simultaneously. While the number of variables grows linearly with the number of parts in the entire assembly, the number of constraints may grow in a linear or a quadratic fashion, depending on the connectivity of the assembly. To reduce the number of equations that are simultaneously solved, Kim and Lee[31] used the relational graph to isolate subassemblies first. Most of the assembly relations are localized within independent groups and part positions are determined locally within these groups.

Turner [37] provided for sequential assembly using relative-positioning operators (RPO). These specialized RPOs are used to determine relative part positions for specific combination of relations. Turner et al. [38] presented a method to obtain the DOF of a part in an assembly from its mating constraints. A *Reduce* algorithm for constraint reduction is presented. A similar algorithm is presented by Kim and Lee [31], but it is limited to planar, cylindrical, and spherical contacts.

In most mechanical assemblies, part positioning is carried out sequentially, with only two parts or subassemblies positioned at a time. Using this strategy, a smaller number of relations, and hence constraints, must be satisfied at each stage even for a large assembly. This can offer significant computational advantage in comparison with a simultaneous strategy. In certain assemblies, it is necessary to position parts simultaneously, but such cases are infrequent. Sequential positioning requires that the assembly do not contain any closed kinematic chains, so that a part can be positioned with respect to parts whose position is completely known. Sequential positioning is possible through a hierarchical position network created by mating conditions. But, mating conditions may exist between parts that do not have a parent-child relationship or the mating conditions that do exist between a parent-child pair may not be sufficient to fully specify the child parts position. Both of these difficulties are addressed in [10].

Scott and Gabriele [39] used a sequential strategy to determine relative part positions. The authors do not linearize rotations, but consider only polyhedral contacts between parts. The designer specifies primary, secondary, and tertiary mating features. The system uses both linear and nonlinear optimization techniques to determine the active surfaces, lines, and points involved in the contacts.

Wu and Kim [40, 41] developed a simple part-mating model by representing the assembly operations as a set of constraint configurations. Based on the unique geometric relationship between two mating parts, 28 constraint configurations are identified to distinguish 17 unique part-mating types. Each constraint configuration is represented by combinations of 3 translational and 3 rotational motion components.

Nnaji and Liu [42] proposed a strategy to represent general product specifications and automatically distribute them among individual components by specifying spatial relationships that describe the desired relationships among components. The spatial relationships among components are used for inferring the assembly position of the components. A rule-based system is developed for assisting the selection of an appropriate set of spatial relationships and for inferring DOF of the assembled components.

Kramer [32] described a coarse sequential-positioning strategy for 2D mechanisms in which geometric relations are satisfied using the DOF associated with bodies. The relations are described as constraints between coordinate systems (markers) on different bodies. For every combination of constraints and the current DOF, explicit motion information is generated to satisfy the constraint. A position solution depends on identifying the DOF associated with each rigid body and the DOF fixed by specified mating conditions between the bodies. This technique is applied to the simulation of multi-link, multi-loop, planar mechanisms. The joint primitives (coincident, in-line, in-plane, etc.) between a pair of markers on rigid bodies are defined which can be used to model all lower and few higher kinematic pairs.

Mullins and Anderson [10] extended the DOF approach to solve positioning problems for mechanical assemblies in 3D with closed kinematic chains. The method relies on a two stage solution process, an initial hierarchical positioning phase followed by a final simultaneous position solution. The mating conditions supported are- against, clearance, flushness, spherical fit, cylindrical fit, and opposition. The constraint equations are formed from the mating conditions and solved simultaneously similar to [7] but with less number of equations and variables.

Kim et al. [43] proposed a 3D constraint solving method for closed-loop assemblies with under-constrained states. They used cut and paste operations on a kinematic joint in a closed kinematic chain to minimize the number of constraint variables that have to be solved simultaneously.

Kim et al. [19] proposed a geometric constraint-solving method that takes well-constrained assembly mating conditions between a base and a mating component and directly transforms them into a 4 x 4 T-matrix. This method is computationally more effective than the numerical or algebraic methods, since the T-matrix is algebraically derived directly from the linear constraint equations associated with the assembly mating conditions.

2.3 Using Group Theory

Thomas and Torras [23] extended Popplestone's [21] work. They find resulting DOF considering intersecting relations among parts. They proposed a general method of finding the configurations of bodies satisfying a set of constraints on the DOF, shape-matching constraints, and non-intersection constraints. In their approach, which is based on theory of continuous groups, relationships such as *fits* are automatically inferred from the shape matching constraints. They represent constraints in terms of chains of symbolic operators that correspond to translation and rotation matrices. They compute the joint effect of two simultaneous constraints by an algebraic reduction of the symbolic expressions. They give a set of nine basic constraint classes, six of which correspond to lower kinematic pairs, and a set of 25 cases for reducing two simultaneous constraints. The constraint equations are generated and solved symbolically.

They also developed an algorithm for inferring feasible assembly configurations from spatial constraints [44]. The process of synthesizing assembly configurations is conceived as a progressive refinement of an initial hypothesis by the application of successive constraints. Two types of spatial constraints: constraints on the DOF between the parts of the workpieces and constraints of non-intersection between workpieces are considered.

Popplestone et al. [45, 46] define a *symmetry* of a part feature as any rigid-body transformation that maps the feature to itself (for instance, any rotation about the axis of a cylinder). They show that symmetries form abstract groups, called *symmetry groups*. Each of the lower kinematic pairs corresponds to a symmetry group. The intersection of two symmetry groups is a symmetry group. They give a list of canonical symmetry groups.

2.4 Using Mathematical Programming

Mullineux [47] tried to automate assembly by formulating various constraints imposed as rules. His approach permits both equality and inequality constraints to be handled in the same way and allows weighting to discriminate between the importance of different constraints. However, it is unclear how the weights can be assigned to achieve functional requirements. A goal function is defined that takes a minimum value when all constraints hold and algorithms are used to numerically seek the minimum. The technique used for incorporating each constraint into the goal function generates a nonlinear and discontinuous function. In using this scheme, unwanted optimum points must be avoided.

Turner [48] proposed a mathematical programming approach in which position of each part is specified based on the geometric relationships between various features of the part and the mating features of its neighboring parts. These feature relationships are treated as inequalities. Mathematical programming is used to find the optimal configuration of the parts. This approach supports both sequential and simultaneous assembly strategies. Because this approach requires linearizing the equations, only small rotational variations are permitted. Only 2D examples are discussed in his approach. Sodhi and Turner [49] extended this approach to 3D. The assembly relations considered are: contacts, attachments, assembly dimensions, enclosures, and alignments. Noninterference constraints are initially generated from contacts, and a feasible region of configuration space is defined within which the other relations are maximally satisfied.

Ge and McCarthy [50] characterized the space of possible relative positions of two components using the Clifford Algebra. The displacements of one part relative to another is identified as elements of the Clifford algebra of 3D projective space. Functional constraints on mating the surfaces of two parts become algebraic manifolds in the vector space of the Clifford algebra. Six primitive constraint manifolds are presented in both parameterized and algebraic form.

Xu and Hannam [51] developed a move-to-contact facility to move an object towards or away from a target object, until they contact or at a given distance from the contact position. This facility enables the separate parts of an assembly to be assembled automatically, and it permits the movement, and potential collisions of the machine assemblies to be checked through simulation. Optimum seeking methods, Bisection, Secant, Curve fitting and extrapolation, and Variable power methods have each been implemented and tested for accuracy and speed of operation, and they have been selectively built into the facility. A multi-axial movement programme has been incorporated to make the methods easy to use and to deal with multi-DOF cases.

Popplestone [22, 52] developed the Edinburgh Design System (EDS), that is based on two types of entities, namely concrete modules and interface module. Each module might in turn have subassemblies or parts. Concrete modules contain parameters, variables, etc. whose values need to be explicitly specified at the time of design (e.g. the diameter of a shaft, and shaft speed). Interface modules encode the constraints between these parameters and variables which form the edges of the graph structure. The representation of assemblies was carried out in terms of a high-level robotic programming language - RAPT [53]. RAPT included only descriptions of the spatial relationships between parts. RAPT is used for input language and a complex AI technique is applied to derive the positions of bodies relative to others given these spatial relationships.

2.5 Using Mating Features

The mating features can also be used to specify the transformations [28]. The particular locations on assembly components are specified to mate. Holland and Bronsvoort [54] define a $feature$ as a physical part of an object mappable to a generic shape and having functional significance. The $assembly\ feature$ is defined as a $feature$ with significance for assembly processes. In the NIST report by Rachuri et al. [55], $assembly\ feature$ is defined as an element to specify the relationships between a pair of assembled components. For example, a hole and a cylinder are typical assembly features. There are a large number of papers available in literature on feature recognition, see Marefat and Ji [56]. Even though all these methods can automatically identify machinable features on a single mechanical component, they cannot be directly applied to the automatic identification of assembly features [57].

Carney and Brown [58] adopt qualitative reasoning approach for feature recognition and mating. It is limited to parts consisting of single base block or cylinder. Each part is considered as a hierarchy of features. Any number of non-interacting features are assumed to be connected to base feature through exactly one face. Features are grouped according to qualitative measures, and then compared to find possible mating. The component rotations are restricted to 90^0 increments only.

Liu and Poppelstone [59] developed a kinematic assembly planning system $KA3$ which reasons about how parts with multiple contacting features fit together automatically from B-rep solid models using symmetry groups. The complete knowledge of the nominal geometry and topology of each body in the assembly and specification of spatial relationships between bodies or between features of bodies are the inputs for $KA3$. It proposes possible assembly configurations and checks the feasibility of each assembly configuration. Only $fits$ contact relation is considered. In [60], they find feasible assembly configurations (FAC) by satisfying both the kinematic and spatial legality constraints from a set of candidate mating feature pairs. In [61], they have described the use of symmetry groups in the analysis of kinematic aspects of assembly.

Driskill and Cohen [62] developed an interactive assembly modeling system based on assembly feature. After the geometry of each part in the assembly, together with its assembly features, has been modeled, the user can interactively put the parts together and perform DOF analysis on them. Assembly feature distributed over several assembly components is handled by defining partial assembly features. It uses data from the assembly planner to determine the order in which to present subassemblies to the user for the interactive specification of connections. At each step, the user selects feature/s to mate on each of the two components, and the system attempts to attach the part to the subassembly in such a way that all the mating conditions are met. If more than one part mating satisfies the required conditions, all possibilities are found, and the user is allowed to toggle through them and select one.

Chang and Perng [12] used high-level entities of mating features and mating relations to describe the composed state of parts in an assembly. The characteristics of volume, reference origin, and boundary face of mating features and parts are used to determine automatic part positioning operations.

Wang and Kim [63] presented a feature based method to obtain assembly mating relations between a set of polyhedral components. Reasoning is based on component geometry only. The components are represented in terms of form features recognized using Alternating Sum of Volumes with Partitioning (ASVP) method. Multiple feature matings is possible. Only containment matings are considered. Parts are assumed to be correctly oriented. All possible assembly configurations can be found.

Sung, Corney, and Clark [57] developed an algorithm using an octree representation of a B-rep model to support the geometric reasoning required to locate assembly features on disjoint bodies. Noort et al. [64] presented an assembly modeling system, in which an assembly model is constructed by specifying the connection features such as pinholes and dovetails.

Ideally, the geometric and dimensional information defined in the feature-based product database could be used to determine physical connections among components in an assembly. Specifically, this could be achieved by matching potential mating features of assembly components. However, this approach is combinatorial, rule based, and may be in conflict with functional specification of product [65]. These limitations led to the development of the relation diagram, where both physical connections and spatial constraints are explicitly provided by the user. Geometric feature information already available in the feature-based product database is used by ElMaraghy and Laperriére [65] to VALIDATE (as opposed to INFER) the physical connections assigned in the diagram by the user.

Qin et al. [66] implemented a concept of drag-and-drop assembly by the introduction of new connection features between two assembly components. The assembly relationships can be pre-described as a connection associated with elementary geometry in modules. Creating an assembly model can be done automatically by mapping the connections, rather than by interactively specifying geometric constraints as in any commercial CAD systems. This would allow non-CAD users to model a machine in a higher level.

3 Assembly Mating Reasoning

Clement et al. [67] describe seven types of elementary mating surface each associated with a unique constraint. The surface resulting from any combination of these seven is also in the set of the elementary surfaces. The lower kinematic pairs are included among the seven types. However, certain combinations of assembly-dimension relations can result in constraints that do not correspond to one of the seven elementary surfaces. They claim that if the relative positions of two parts are constrained by two or more of the lower kinematic pairs, then the net effect may be reduced to a single constraint. Either this constraint corresponds to one of the lower kinematic pairs, or the two parts are rigidly attached.

Morris and Haynes [68] presented a robotic assembly system, "Assembly by Constraints (ABC)", in which the assembly relations are characterized on the basis of the relative DOF constrained. Since two parts have six relative DOF, there are 26 combinations of these six DOF. Of these, the redundant and unrealizable combinations are removed to get a set of 15 ways in which to constrain the six relative DOF. These 15 relations are then used by the designer to define the assembly. However, the possible relative motion between two subcomponents in the assembly is not explicitly specified.

Srikanth [69] described the use of the de Mello and Sanderson's representation [70] to determine enclosure and alignment relations from contact information efficiently. It is extended to include non planar contacts, with the assumption that the contacts are complete, that is, they extend a full 360^0 about the axis or center point.

Thomas and Nissanke [71] presented an assembly representation scheme based on attribute graphs for describing assembly parts and an algebra to model assembly tasks. Any task is modeled as a merging of two attribute graphs to produce a single attribute graph which represents the merged or assembled object. The assembly tasks involving only circular right cylinders are considered. The contact types are limited to cylindrical hole-shaft, threaded hole-shaft and screw pair.

Mullins and Anderson [5] presented techniques for the automatic identification of assembly mating constraints in computer models of 3D assemblies with nonorthogonal contacts between component surfaces and kinematic joints. The approach relies on a graph-based representation of the assembly. The concepts of constrained groups, physically constraining face set, and characteristic vector space are introduced for identifying and representing assembly constraint relations. Graph based algorithms are described for identifying and manipulating the constraints based on geometric reasoning and kinematic analysis.

4 Deriving Kinematic Information

Downstream applications such as tolerance and mechanism analysis are very useful to validate or refine a product design. The lack of a complete mathematical description in a conventional system severely restricts its ability to use a common model for design and analysis, which necessitates special expertise to rebuild analysis specific models. The model-rebuilding process is very time-consuming and error-prone [72]. The various approaches developed to derive the kinematic information for motion analysis are discussed in this section.

4.1 From Mating Relations

Kim and Lee [31] described a system for kinematic analysis in which the joint information is automatically derived from the mating relations for each link. The system considers only planar, cylindrical, and spherical contacts. They claim that the methodology infers revolute, prismatic, cylindrical, planar, and spherical joints from against, fits, and spherical fits mating constraints. For these relations, a set of rules is generated that compute the resultant DOF at the joint. However, the use of such a rule-based approach to determine the combined effect of multiple relations is somewhat ad hoc. New relations cannot be supported unless rules are provided for them also. The automatic positioning of components in the closed loop assembly is not correct, and requires the user intervention to find the intended configuration. The component's position is derived from mating relations by constructing the constraint equations and solving them simultaneously using Rocheleau's approach [29].

Kim and Wu [40] presented a classification of various mating constraints from the perspective of the DOF that are constrained. DOF are identified before, during, and after the assembly operation.

Turner et al. [38] described a scheme to represent every relation in terms of the DOF that are constrained. A procedure to determine the combined effect of two or more relations, and the use of this procedure in identifying pairs of parts that have no unconstrained relative DOF is described. This information is subsequently used in the generation of a hierarchical model of the assembly. They claim that it is possible to treat translational and rotational DOF independently. However, in some examples interactions exist between translational and rotational DOF. This approach do not correctly identify the DOF associated with an assembly component because it is assumed that specification of mating constraints automatically constrains the component to maintain the required contact relationships [73]. The separating motions are not considered.

Kramer [32] determined the effect of geometric constraints on the DOF using a lookup-table procedure. The current DOF and the geometric constraint are inputs, and the resultant DOF and motion to satisfy the constraint are the outputs. The table has approximately 100 entries.

Mattikali and Khosla [74] described a method to determine the kinematic DOF associated with component mating constraints. The work is an extension to 3D space of the method presented by Reuleaux [1] for planar contacts. The space of all available DOF is represented using a unit sphere. The translational DOF remaining after applying surface mating constraints are identified by superimposing restraint hemispheres caused by individual planar contacts. For rotational DOF, constraint spheres are identified for each surface contact. The unconstrained DOF are identified by superimposing these spheres. They have worked with polygonal bodies and polygonal surfaces of contact. Curved planar boundaries are approximated using straight lines, and linear programming techniques are used to solve the contact problem. While the methodology is general for various types of mating constraints, and identifies all available DOF, the procedure is complicated and the likelihood of rotational DOF about arbitrary axes is small in mechanical systems.

Mattikalli and Khosla [75] presented a method to determine constraints on translational and rotational motion of planar and 3-D objects from their contact geometry. Translations are represented by spatial vectors and rotations by axes in space. A geometric representation of the space of all possible motion parameters is constructed. The restrains imposed by a single mating surface element is analyzed by determining portions within this space that are disallowed. By computing the union of the disallowed regions due to each contact surface element, and taking its complement with respect to the whole space, the space of allowed motion parameters is obtained. Rajan and Nof [76] also employed the similar idea to define the mating direction set. Wilson and Latombe [77] introduced the concept of a non-directional blocking graph (NDBG) which describes the allowed DOF after surface mating constraints have been considered.

Nakamura and Nakajima [78] presented a method for kinematic model extraction from a given mechanism. Shapes and composition of machine components are described and input to the computer using Feature Description (FD) language. Feature Description language is a verbal description of shapes, dimensions, and mating surfaces of the elements of machine. This data is stored in a composition diagram which is successively updated. From this, contact list is deduced, indirect contact relations analyzed, mutually unmovable parts set to single part, and then kinematic pairs are extracted to deduce the kinematic model. Only planar, cylindrical, screw, and gear contacts are considered. Additional constraint on contact plane normal, cylinder, screw, or gear axis is put to be parallel to Cartesian coordinate axis which may not be the case always. Rules are formulated based on contact conditions to extract kinematic pair. The extracted kinematic model as a result is presented using a graphical expression named Composition Diagram.

Prinz et al. [79] developed Product Assembly Modeler (PAM). It is a CAD based kinematic modeling system which associates the kinematic modeling with assembly modeling processes, such that, constraints to kinematic pairs are automatically generated when components are appropriately assembled. The kinematic analysis is done by solving selected set(s) of constraints. The geometric and topological information in the system's database is utilized to derive and solve the kinematic equations that characterize a mechanism. The user specifies the relative positions of the components in the assembly by defining spatial relationships between the features of components. The list of spatial mating constraints [21] is expanded in this work. The DOF derived from the mating constraints is similar to those proposed by Turner et al. [38], and therefore does not consider separating motions. Also, it is not clear how offset and parallelism constraints can affect joint kinematics. PAM employs a rule based spatial relationships engine to infer the DOF associated with a kinematic pair. For closed loop mechanisms inverse kinematic analysis is used to find initial configuration. Loop closure matrix equation is formed and solved by Newton-Raphson iterative procedure.

Anantha et al. [34] described an assembly modeling system in which a bottom-up process is used. Components are modeled using a feature-based design module with a limited set of machining features. Structural primitives (top, bottom, side, and end) are assigned to the features. Fits, against, and parallel mating constraints are then used to define relationships between the mating features. These mating constraints along with the feature geometry are used to determine the joint kinematics. The constraints are solved incrementally by symbolic geometric reasoning called DOF analysis. Similar to the work by Turner et al. [38] the approach assumes that specification of a mating relationship implies forced contact between components, which is incorrect.

Rajan et al. [80] developed an algorithm that identifies the set of unconstrained DOF by considering the various types of mating constraints. It enumerates parts that can separate to determine the component DOF. This algorithm is implemented in OpenADE [81] to propagate and verify the kinematic design intent as the design progresses through each design stage and across different tools. OpenADE uses an icon-based representation to display the component DOF to the designer. This visual feedback helps the designer identify under- and over-constrained components. Furthermore, this approach avoids redundant and potentially conflicting specification of kinematic joint constraints because it automatically computes this information from the assembly mating constraints. The prototype implementation is limited to interpreting *against* and $fits$ constraints associated with planar and cylindrical contacts.

4.2 From Contact Conditions

A part in an assembly is in physical contact with one or more other parts. Some of these contacts induce surface mating constraints, leading to the formation of a joint. Other contacts are incidental, in that they may introduce limits on the DOF of the joint. Reasoning about these constraints provide the designer with valuable insight into the instantaneous DOF of the assembly [76]. Reuleaux [1] analyzed the effect of point contact on translational and rotational motion of planar objects. Using a graphical method, he derived the field of restraint for a point contact. Ohwovoriole [82] used the theory of screws to study the kinematics of the relative motion of contacting bodies. In dealing with planar motion, Reuleaux's method is simpler than that of Ohwovoriole and provides identical results. Mattikalli and Khosla [75] extended Reuleaux's method to spatial motions of 3D objects. The analysis of restraints is presented for point contacts as well as the contact extending over surfaces.

Sinha et al. [83] proposed a methodology to determine the instantaneous DOF of an assembly, given the assembly with the geometry and positions of its components. The nature of body to body contacts is analyzed to obtain surface mating conditions. Simultaneous satisfaction of the non-penetration conditions at all the contact surfaces between a pair of bodies is represented by a 6D simplex, which can be solved using linear programming. Knowing the instantaneous DOF, a point and line articulation representation is obtained. Only with planar, cylindrical, and spherical contacts forming the lower kinematic pairs are considered. It can handle incomplete geometry such as portions of planes, cylinders, or spheres. Some post processing is required to eliminate multiple identifications of same DOF. This work is generalized by Sinha et al. [84]. They have obtained a set of properties that must be satisfied by a general contact surface in order to preserve the linearity of the model. The feasible joints are found from the space of allowable motions using heuristics. The method is algebraic and uses linear programming. It is relatively fast and is valid for all possible surface contacts, unlike rule-based systems that operate on a feature level. The algorithms can find only one configuration of a mechanism.

Gupta et al. [85] developed an Intelligent Assembly Modeling and Simulation (IAMS) environment. The assembly representation developed allows to capture the articulation in assembly. Feature recognition techniques are used to recognize the joints between parts and assembly features on individual parts. Beginning with the set of part geometry, all the possible contacts between these parts are generated, and an undirected contact graph is populated. The contacts can be classified into two types - constraint contacts and incidental contacts. Some non legitimate joints identified from contacts has to be interactively deleted by the designer. The kinematic joints considered are fixed, revolute, cylindrical, prismatic, planar, and spherical.

5 Kinematic Modeling

5.1 Configuration Space Approach

The configuration space (CS) of a mechanism defines the set regions of free placements of the objects so that no two objects overlap. The type (topology) of the free placement region determines the type of object motion that can occur in it. The connections between the regions determine the possible behavioral transitions. The actual behavior of a mechanism, resulting from specific input motions applied to its parts, corresponds to a one-dimensional continuous path through the CS [86]. CSs are an appropriate representation for relating kinematic behavior and object geometry for mechanism analysis [87, 88]. This approach respects the physical reality by considering the geometric extent of the parts.

Joskowicz [87] designed a two-part algorithm for predicting the behavior of mechanical devices. The first part of the algorithm finds the possible relative motions of all pairs of objects initially in contact. First, the pairs of objects are classified into lower and higher kinematic pairs. A rule based approach using predefined CS groups is used to identify lower pairs [89]. For predetermined higher pairs (e.g. gears) the possible motions between parts are validated against predefined conditions. For unknown higher pairs, the differential behavior of two components is deduced from the rules and integrated to find the behavior over a period of time. The second part of the algorithm takes the possible motions of each pair of contacting components and the input motion, and determines the behavior of the entire mechanism. This algorithm is applicable only to fixed axis mechanisms.

Joskowicz and Sacks [90] developed a kinematic analysis algorithm for fixed axes mechanisms built of rigid parts. The algorithm optimizes the computation by decomposing complex mechanisms into subassemblies, deriving the kinematics of the subassemblies, and incrementally composing the results. The inputs are the shapes and initial configurations of the parts. The output is a region diagram, a partition of the mechanism configuration space into regions that characterize its operating modes. This approach can handle permanent as well as intermittent contacts.

Sacks and Joskowicz developed kinematic analysis algorithms for fixed-axes planar higher pairs [91]. Sacks further developed it for general planar pairs [92] based on CS computation. This work is extended by them with Kim [93] for spatial fixed axis higher pairs.

5.2 Symmetry Based

Gelsey [94] developed algorithms for automatic reasoning about machines. A behavioral model of a machine is created directly from a Constructive Solid Geometry (CSG) representation of the physical structure of the system to be modeled. The lower kinematic pairs are identified from the common symmetry between the subparts (primitive solid) in contact. The only higher pair that can be identified is a gear pair.

5.3 Screw Theory Based

Konkar and Cutkosky [95] created screw system representations of assembly mating features and used screw theory to determine the number of relative DOF between parts in an assembly. Adams and Whitney [96] used Konkar's algorithm and extended his work by defining extensible screw representations of many types of assembly features. Screw Theory is used to provide mathematical models of assembly features, allowing the determination of positioning constraints imposed on one part in an assembly by another part based on the geometry of the features that join them. A user of this theory is able to combine members of this set to join two parts, and then determine whether or not the defined feature set over-, under-, or fully-constrains the location and orientation of the part. Motion Limit Analysis (MLA) [97] uses the mathematics of screw theory to model the ability of mechanical assembly features to allow or constrain rigid body motions in six DOF. The directions and quantitative amounts of possible finite rigid body motions of a part that is being added to an assembly can be determined via calculation applied to a defined set of assembly features.

5.4 Port Based

An *assembly port* [98] is defined as a group of one or more low-level or basic geometric entities, such as faces, edges, or centerlines, that undergo mating constraints in order to join parts in a CAD assembly. An assembly port (e.g. peg, hole) comprises one or more low level geometric entities that belong to the same part unlike assembly feature (e.g. peg-hole) connecting form features from two different parts.

Cutkosky et al. [99] described a concurrent design system that supports various stages of the design process. An assembly is represented as a graph consisting of components and their interactions. These interactions impose certain constraints on the mating relationships between the components. The components have ports that define standard connectivity relationships. Standard components such as bolts, and nuts have pre-defined ports, but the user can define ports on new components. Ports are further decomposed into component features such as holes and protrusions, which are further decomposed into lower-level geometric information. The approach used is to derive lower-level geometric and kinematic information based on higher level connection definitions.

Tiihonen et al. [100] developed the Ranger Configuration Model and applied the concept of *Port Type* as a connection interface and a *Port Individual* as a *place* where a component may be connected. A port type has a compatibility definition that defines a set of port types to which it can be connected and a set of connection constraints. The configuration model validates the assembly interfaces only for a particular assembly and not for all assemblies.

Sinha et al. [101, 102] developed a framework that can automatically derive the behavioral models of the components from the geometry. The concept of combining both form (CAD models) and behavior (simulation models) of mechatronic system components into component objects is introduced. This framework is integrated with CAD, by providing algorithms to extract the type and parameters of a lower pair from the geometry of the interacting components. To achieve the composition of behavioral models, a port-based modeling paradigm where systems consist of component objects and interactions between them is introduced. The constraint is imposed on the port variables of the rigid bodies and is represented by a joint component with two ports. The behavioral model of the joint component relates the variables of the two ports and captures a kinematic or dynamic relationship between the components. The joint types that are supported include the lower pairs, gear pairs, and rack-and-pinion.

Singh and Bettig [98] evaluated and compared different schemes for capturing the attributes of assembly interfaces and appending that information to solid models. For evaluation they introduced the concept of *assembly ports*, which are defined as a group of one or more low-level geometric entities that undergo mating constraints in order to join parts in a CAD assembly. The geometric entities participating in connections (assembly ports) are explicitly identified and labeled in such a way that only valid connections are allowed so that standard components can be assembled automatically in novel configuration designs.

The port concept is useful for automated generation of assembly design alternatives, and it is also beneficial to CAD assembly designers in reducing the number of steps required. For assemblies that involve non-standard or one-off parts the time spent defining ports on them is wasted. The system developed by Singh and Bettig [98] seamlessly allows applying mating conditions manually on such parts and using ports on others.

5.5 Feature based

Johnson [103] developed a new style of kinematic analysis software which is directly accessed by another calling program, rather than via a user interface. He considered only right circular cylindrical holes and shafts forming kinematic pairs. Cylindrical holes and shafts are divided into four categories based on the features (groove, circlip) present on them. The kinematic pairs between bodies are not specified explicitly by the user, but are defined implicitly from the relevant features of the two parts involved in the pair. Each hole and shaft is marked with four vectors. Seven types of joint primitives are defined using these vectors. The combinations of shafts are analyzed for the joint primitives and kinematic pairs are identified from them. Three procedures $ALIGN$, $CROSS$, and $SETSQ$ are given to find the assembled configuration. The position analysis is done using constraint matrix. For small adjustments in position and orientation of parts, a closed form solution is given. A modified Newton-Raphson method is suggested for the large adjustments. The rotation of body is restricted up to 45^0 to avoid unstable iterative behavior of numerical procedure.

The DOF concept is used by Eng et al. [104] to characterize the kinematics of an assembly process. In particular, the DOF between the features of two mating components are established. They are used to construct the feature matrices, which are in turn used in setting up the kinematic pair liaison diagram (KPLD). Intersection iteration among all the features in a component generates its overall DOF.

Kikkawa et al. [105] developed a mechanism modeling system to support the top-down design process. The mechanism is presented by constraints and default values because constraints are suitable for representing the abstract model for a mechanism and default values are useful to supplement an incompletely defined model. The module-based modeling with mechanism modules corresponding directly to the functionalities of the mechanism is introduced. Incomplete constraints are solved with default values so that the shape and location of the incompletely defined model can be modified, and the motion of the model can be simulated.

An Extended Variational Design Technology (VGX) developed at SDRC [72] can perform mechanism analysis using assembly constraints and dimensions defined within variational assembly models. This capability allows an engineer to quickly evaluate the kinematics and dynamics of a mechanism without the burden of entering redundant information, such as joint definitions, in a format that mechanism analysis packages can understand. Furthermore, there is no need for rebuilding the mechanism models after making design changes to the models.

6 Discussion

Many assembly modeling techniques in literature and all the current commercial CAD packages are based on the the explicit specification of the mating conditions by the user. But, the same assembly configuration can be achieved by specifying different sets of mating conditions. Hence, it is not consistent. It is possible to specify redundant (not reducing the available DOF) mating conditions that makes assembly over constrained. The geometric entities like center points, center lines used in assembly mating conditions are not physically existing on the parts. The use of such abstract geometric entities is non intuitive. The geometric entities involved are the idealized mathematical representations of the actual geometries of the part e.g. a planar face with a definite boundary and an outward face normal is considered as an infinite plane with two sides. Similarly, a straight edge is considered as an infinite straight line without any convexity information associated with it. In conclusion, while defining and satisfying mating conditions on the geometric entities of the parts, the actual extent of the geometry is not taken into consideration. User has to select a proper pair of geometric entities to apply a mating condition and should know its effect on the relative motion between the two parts.

Geometry alone does not conveniently provide information about how parts in an assembly might be connected. It is very difficult, if not impossible, to determine whether a particular indentation in a part geometry is there to make the part lighter or whether it is intended to mate with a protrusion on another assembly component. However, since the designers create the forms with ceratin identified portions of the parts to be in the mating configuration, derivation of assembly possibilities should not be regarded as a pure jigsaw puzzle. Attainment of these configurations physically depends upon the mutual geometric forms and their dimensional compatibility.

Two parts can be placed in space arbitrarily so long as their physical extent do not interfere with each other. Parts assembled with mutual contact can maintain their position relative to each other even in presence of gravity. Description of this static relative configuration has been tried in literature using diverse methodologies in assembly modeling. If the geometric constraints are specified improperly, it may not be possible to resolve the type and location of contacts between parts. Hence, the stability and equilibrium of the parts may not be determined. Thus explicit specification of relative part positions require expertise and in depth understanding about necessary and sufficient conditions to be specified.

On the other hand, when two parts are in contact, it is possible that one part move with respect to the other without losing contact although location and number of contacts could change without affecting the relative motion. Certain combination of shapes of parts especially where they are in contact with other parts allow certain characteristic relative motion to be identified as a kinematic pair. Explicit specification of the kinematic constraints may or may not obey this physical reality. This may cause the conflict between the assembly and kinematic constraint specifications. Thus, specification of kinematic constraints require domain expertise and knowledge about necessary and sufficient degree of constraints to be specified. Thus an in depth understanding about contact alone can implicitly determine relative part position and relative part motion simultaneously without the need of explicit specification of assembly or kinematic constraints.

7 Conclusions

The specification of constraints on the relative motion between the parts is not sufficient to completely define their positions but the relative motion can be inferred from the part positions and their geometry.

Current standardized representations of assembly information do not contain all the data needed in assembly related design applications. The missing data includes assembly level tolerances, assembly mating constraints, and component kinematics information.

User-defined articulation is time consuming and prone to errors. Many representations are tool-specific (ADAMS, Pro/ENGINEER, etc). One has to model the part geometry and the mating surface constraints in the same CAD tool; model export or model translation does not retain articulation information.

Both part-positioning methodologies used in assembly modeling and kinematic pairing methodologies used in kinematic modeling mostly aim at capturing the state of interaction between two parts arising out of the physical contact between the parts. Whereas the first one aims to describe "where a part is" in space, the second one aims to describe "where the part can move" in space. These two problems have a dual relationship.

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